Optimal pricing for shared vehicles

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Abstract

The market of shared vehicles (SV) has a unique feature: the key product characteristic, location of vacant vehicle, is determined by previous customer rather than provider. This paper offers a model of SV market, with particular emphasis on spatial inequality of demand. Using recursive methods, I show that optimal pricing policy assigns a score to every location, and rewards (penalizes) customers for relocating the vehicle to a place with higher (lower) score. Optimal pricing enables providers to expand service into low-density suburban areas. The theoretical model is fitted to novel microdata on SV trips in Moscow, Russia. A counterfactual exercise shows that a hypothetical new operator using optimal pricing, by expanding the service area to suburbs and charging monopolist markups there, can increase gross profit four times relative to existing competitors.

Keywords: Shared vehicle, Home area, Spatial pricing, Trip demand

JEL codes: C53, L91, R41

1. Introduction

Shared vehicle (SV) service is a rapidly growing industry with millions of customers around the world. Customers of such service typically book a vehicle of choice via a mobile app, walk to the vehicle, drive it while paying per minute, and drop it off at a permitted location. Vehicles are typically not serviced between customers and are available to other

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customers immediately after drop-off. This paper is primarily focused on *free-floating* vehicle sharing, which allows drop-off at any legal parking location within a predetermined area, although the results of the paper can also be extended to other supply modes.

Shared vehicles, essentially a per-minute vehicle rental service, should not be confused with ride-hailing, essentially a taxi service. Shared vehicles are cheaper than ride-hailing because the customer does the driving. A key feature of SV business model is very high cost of relocation of vacant vehicles, which makes this service viable only if a vehicle dropped off by a previous customer is expected to be booked by next customer at the same location within reasonable period of time. This feature leads to a key drawback of existing SV services: their geographic coverage remains limited to areas with very high density of business actvity. For example ShareNow, the world's biggest provider of free-floating shared automobiles, serves only 17 large European cities; even in those cities, the service area is typically limited to high-density urban core. In Russia, two-thirds of available shared cars are based in Moscow, which accommodates only 8% of country population. Operators typically exclude low-density areas due to expectation of low vehicle turnover rate.

This paper develops a theoretic model of SV market, and specifies an optimal pricing policy that allows to expand vehicle sharing service virtually to all locations with any potential travel demand. The model features arbitrary geography and spatial inequality of travel demand, as well as arbitrary coverage area and pricing policy by competing operators. In addition, the model features arbitrary per-minute parking fees that shared vehicles are charged, which enables operators to expand drop-off area to private parking lots.

The above-mentioned feature of SV service, that relocation of vacant vehicles is economically impractical, implies that vacant vehicle location is determined by the previous customer. This feature invites the use of recursive methods typically used in macroeconomics, which I do in this paper to formulate profit-maximizing pricing schedule. There are two key theoretical findings that current SV operators are apparently not using.

First, I find that optimal price is a concave function of trip duration: the rate for initial few minutes of the trip should be as high as a monopolist price, while the rate for further minutes should decrease to marginal cost level. Existing operators typically offer a flat per-minute rate, only allowing discounts for very long rental durations.

Second, I find that every pick-up/drop-off location should be associated with a location *score*, with higher score typically corresponding to higher-travel-demand locations. Each customer, in addition to conventional per-minute rate, is charged the difference between the pick-up and drop-off location scores. Thus, customers traveling from city centers to suburban areas are typically charged an extra fee, while customers traveling in opposite direction earn a bonus of the same amount.

My concept of location scores is closely related to the concept of location *potential* by Koopmans (1949) who studies optimal pricing for an unbalanced transportation market, i.e. market where inbound transportation demand for a given location is unequal to outbound demand. Like the current paper, Koopmans (1949) specifies a score/potential for every location, so a customer, in addition to usual fees, pays the difference between origin and destination potential. This means being rewarded for demanding travel against the main traffic, and being penalized otherwise. While the current paper in most general form features the same demand imbalances, I focus on another type of spatial heterogeneity which is highly relevant for the SV market: some locations having higher overall travel demand than others, leading to heterogenous expected times of vacancy (i.e. of waiting for next customer). This dimension is omitted in the static model of Koopmans (1949) who assumed zero waiting times; I show that such heterogeneity alone, even with fully balanced inbound/outbound travel demand, leads to heterogenous optimal location scores.

In addition to theoretical contribution, this paper fits the model to novel data on car-sharing trip demand in Moscow, Russia, featuring over 300 000 trip observations. A counterfactual exercise simulates a hypothetical new operator that uses theoretically informed pricing and competes with existing flat-rate operators. The new operator serves all suburban locations with any potential for travel demand, an area four times as large as that of existing operators. To predict travel demand to/from areas currently not served, I use a novel technique of fitting the observed travel demand to night light data, with high goodness-of-fit. Because the new counterfactual operator is a monopolist in the newly served areas, it can charge monopolist markups there, which is shown to increase gross profit per vehicle fourfold relative to that of incumbent operators.

To the best of my knowledge, no economics journal has previously published a theoretical study of industrial-organization aspects of shared vehicle service. There are some studies of related taxi and ridesharing market. Lagos (2000) recognizes the spatial frictions of matching riders to taxi cabs, and develops a theoretical search model of this market. Lagos (2003) takes this model to New York City data; Frechette et al. (2019) is a similar empirical study of New York taxi with newer data, temporal variation in demand, and explicit accounting of entry barriers. None of these studies, however, consider a change in the service area or optimal pricing.

In the transportation literature, there is a substantial number of studies on how to maintain a desirable distribution of vacant shared vehicles within a given service area. Zhang et al. (2019) propose introduction of negative prices to encourage optimal rebalancing of shared bicycles. Wagner et al. (2015), in a counterfactual analysis of a free-floating SV service, study the effects of offering customers a small fixed bonus for altering their destination location by few hundred meters, towards a spot more desirable for the operator. Lippoldt et al. (2019) empirically analyze the effects of such policy implemented in practice. These studies do not question optimality of the existing service area; moreover, the relocation bonus of the latter two studies in not theoretically informed. In particular, it is not clear why incentivizing a customer to change trip destination from point A to point B is any better than incentivizing the next customer to change trip origin from B to A.

Several simulated models of an SV market investigate the effects of various counterfactual policies. Ciari et al. (2015) study the effects of various pricing schemes on SV demand; their menu of pricing schemes, however, is not theoretically informed and not location-specific. Balac et al. (2017) argue that increased parking rates will induce travelers to switch from personal to shared vehicles, as the latter typically have shorter parking durations.

There is empirical literature analyzing various aspects of free-floating SV service. Becker et al. (2017) study the determinants of SV choice, as opposed to other transportation options, using data from Basel, Switzerland. Ampudia-Renuncio et al. (2020) analyze the spatial distribution of SV trips in Madrid.

Another strand of literature studies social impacts of SV service, dating back to McCarthy (1984). Mounce and Nelson (2019) is a recent example of this line of research.

2. The model

2.1. Time and space

Define by *zone* a geographic area such that any two points within the area are of walking distance from each other, e.g. a residential neighborhood or a couple of blocks in the central business district of a city. Within a zone, all events occur in a single-dimensional space in which all locations are ex-ante identical to each other; one can think of each zone as being a circular street.

The world consists of a set $\mathcal{I}$ of zones that are sufficiently distant from each other, so that vehicles are demanded for travel between them. One of transportation options is an SV.

Time in the model is continuous and is measured in minutes. For transparency of the main results, we will assume that all parameters in the model are time-invariant, thus only steady states will be analyzed.

2.2. Travel demand

Emergence of potential customers in each zone i willing to travel to another zone j by SV is a Poisson process, such that the expected number of potential customers emerging during a time interval $\mathrm{d}t$ within a segment of space $\mathrm{d}l$ is equal to $\lambda_{ij}\mathrm{d}t\mathrm{d}l$. We will nickname λ_{ij} as *demand density*. The exact destination location within the destination zone is random, uniformly distributed across space, and is independent from the origin location. The ride between the two zones takes h_{ij} minutes, regardless of the exact locations of origin and destination.

I assume every zone i has strictly positive outbound $\sum_j \lambda_{ij} > 0$ and inbound $\sum_j \lambda_{ji} > 0$ travel demand. Furthermore, I assume that travel demand satisfies the *connectedness* condition: the set $\mathcal{I}$ of all zones cannot be partitioned into subsets $\{\mathcal{I}_1, \mathcal{I}_2\}$ such that $\lambda_{ij} = 0$ for every $i \in \mathcal{I}_1$ and every $j \in \mathcal{I}_2$. In other words, the SV service area cannot be split into two areas with zero travel demand between them.

It is quite plausible that customers' value of travel v is proportional to h_{ij}, as the cost of their outside transportation options (e.g. of taxi) likely increases with travel time. I assume that customers have a heterogenous v, distributed exponentially with mean θh_{ij}, where $\theta > 0$ is mean value per minute of travel: $\Pr(\text{value} > v) = \exp(-\frac{v}{\theta h_{ij}})$.

Vacant vehicles are scattered along space within each zone. As the space within a zone is continuous while vacant vehicles are discrete, customers will almost surely need to walk to the SV of their choice. We assume that walking incurs a dollar-valued disutility w per unit of distance. The walking speed is normalized to unity, hence the distance x between the customer and a vehicle is also equal to the walking time. During this time, the vehicle must be reserved by the customer and unavailable for alternative uses.

As the space at the destination zone is homogenous, all customers are always better-off travelling directly to the destination point, and no walking costs are incurred upon arrival. The vehicle becomes available to other customers immediately after the end of the trip. Given

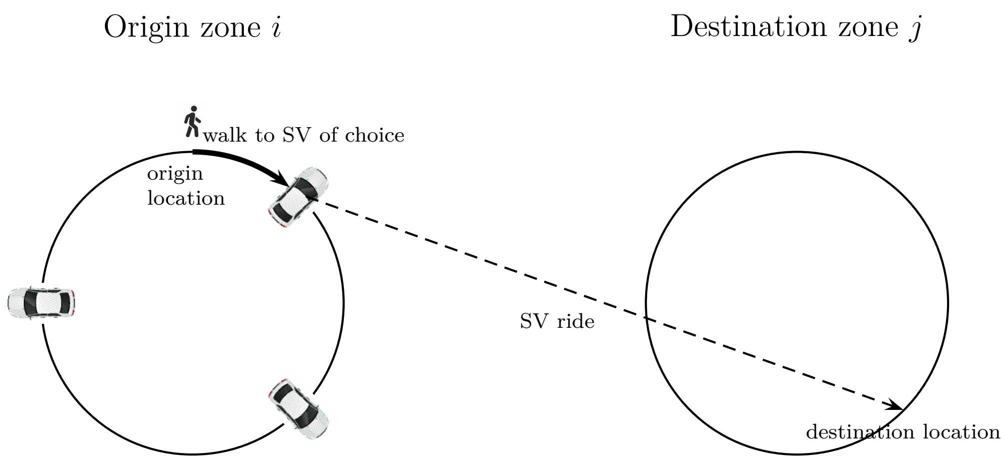

Figure 1: Illustration of a typical trip. Vehicle image courtesy of Macrovector/Freepik.

random and uniform distribution of trip destination locations within each zone, vehicles available to the next customer are distributed uniformly across space. Figure 1 illustrates a typical trip by SV. All customers have an outside opportunity of zero: if the value of travel v is below monetary and walking costs detailed below, a customer will not use SV and take the outside opportunity (e.g. another transportation method).

The total price p_{ij} for renting a vehicle may generally depend on the walking time x between the customer's origin location and the vehicle, due to the need to reserve the vehicle during this time. Then, the customer selects the vehicle k that minimizes the total cost of the trip $p_{ij}(x_k) + wx_k$. As elaborated in the previous paragraph, no vehicle is chosen if $v < \min_k\{p_{ij}(x_k) + wx_k\}$.

2.3. Travel supply

The supply side of the market is presented from the perspective of a typical SV company, *the operator* henceforth, facing competition with other similar operators. The operator has a fleet of SVs such that, in the steady state, the density of its vacant vehicles in zone i is μ_i, i.e. on average there are $\mu_i \mathrm{d}l$ vacant vehicles in the space of length $\mathrm{d}l$. Because other operators may have different geographic coverage of service, the density of vacant vehicles by other operators is destination-specific and is denoted $\tilde{\mu}_{ij}$. For example, if some other operator allows its vehicles in zone i to travel to zone j but not to k, then such vehicles are included into the calculation of $\tilde{\mu}_{ij}$ but excluded from $\tilde{\mu}_{ik}$. In particular, $\tilde{\mu}_{ij} = 0$ if no other operator is serving zone j, meaning that the operator in question is a monopoly on that route.

The price per trip chosen by the operator is a non-decreasing function of walking time $p_{ij}(\cdot)$. Other operators are assumed to have a pricing policy linear in the walking time: $\tilde{p}_{ij}(x) = \tilde{p}^a_{ij} + \tilde{p}'_i x$. For notational simplicity, $\tilde{p}_{ij}(\cdot)$ is identical across all other operators. Such assumption appears to be empirically relevant, for example pricing policies of different carsharing operators in Moscow are very similar to each other.

The operator costs associated with each vehicle include:

- Movement costs (fuel, cleaning, maintenance, depreciation, etc.): c per minute, ch_{ij} per $i \to j$ trip;
- Parking costs of g_i per minute in zone i. The parking cost is paid when the vehicle is vacant or reserved;
- Fixed costs: vehicle lease or loan payments, insurance, and any other expenses unrelated to how the vehicle is used.

Given dynamic nature of the environment, for a given fleet of vehicles the operator maximizes its stream of *gross profit*, equal to the stream of revenue minus movement and parking costs. As rental periods tend to be very short, measured in minutes or hours, there is no discounting of future periods: gross profit earned at 1pm has the same value to the operator as gross profit earned at 5pm of the same day. Instead of *discounted* stream of profit commonly used in economics literature, I will assume that operators maximize the *average* gross profit earned per minute per vehicle, denoted ϕ and introduced formally in section 2.5.

I assume free entry of new operators, meaning that the average flow of gross profit ϕ per vehicle must equal the flow of fixed costs in equilibrium.

2.4. Vehicle demand

Given spatial density of vacant vehicles μ_i, the probability that a customer walking in a certain direction will reach a vacant vehicle within distance x is (dividing space $[0,x]$ into segments of infinitesimal length Δ) $\lim_{\Delta\to 0} 1-(1-\mu_i\Delta)^{\frac{x}{\Delta}} = 1-\exp(-\mu_i x)$.

We now calculate the probability $D_{ij}(x, p_{ij}(x))$ that a customer originating her trip in zone i at distance x from a specific vehicle k and destined to zone j will indeed book this vehicle. Such outcome will take place if all of the following three independent events occur:

1. The customer's value exceeds the travel cost, $v > p_{ij}(x) + wx$, which occurs with probability $\exp(-\frac{p_{ij}(x)+wx}{\theta h_{ij}})$.[2]

2. No other vehicle of the same operator is more proximate to the customer. As the customer can walk in two directions, there should be no other vehicle within range $[-x, x]$ from the customer's position, which occurs with probability $\exp(-2\mu_i x)$.

3. Vehicles of other operators are distant enough. As trip costs of other operators may differ, denote by $z_{ij}(p_{ij}(x))$ the maximum distance between the customer's origin and some alternative vehicle more attractive than k: $z = 0$ if $\tilde{p}_{ij}(0) \geq p_{ij}(x) + wx$, and $\tilde{p}_{ij}(z) + wz \equiv p_{ij}(x) + wx$ otherwise. Linearity of $\tilde{p}_{ij}(\cdot)$ allows us to express z explicitly:

$$z_{ij}(p_{ij}(x)) \equiv \max\left\{\frac{p_{ij}(x) + wx - \tilde{p}^a_{ij}}{\tilde{p}'_i + w}, 0\right\}. \quad (1)$$

   Vehicle k will be taken if there are no competing vehicles by other operators within range $[-z_{ij}(p_{ij}(x)), z_{ij}(p_{ij}(x))]$ from the customer's position, which occurs with probability $\exp(-2\tilde{\mu}_{ij} z_{ij}(p_{ij}(x)))$.

[2]Note that if $p_{ij}(x)$ is negative, i.e. the operator is paying a bonus to its customer for the trip (which is relevant in some applications below), this probability may exceed unity. While mathematically incorrect, this feature proxies for the fact that negative prices may attract individuals who have no intrinsic value of an $i \to j$ trip, ride the vehicle only to collect the bonus, and thus were not included into the calculation of demand density λ_{ij}.

Given the above calculations, the probability $D_{ij}(x, p_{ij}(x))$ is given by

$$D_{ij}(x, p_{ij}(x)) = \exp\left(-\frac{p_{ij}(x) + wx}{\theta h_{ij}} - 2\mu_i x - 2\tilde{\mu}_{ij} z_{ij}(p_{ij}(x))\right). \tag{2}$$

Note that $z_{ij} = 0$ in (1) only if $p_{ij}(0)$ is lower than $\tilde{p}_{ij}(0)$ and only for sufficiently small x. To streamline the exposition of the results that follow, we will ignore the zero lower bound on z. This will cause low-price operators to overestimate competition for customers in close proximity of their vehicles. This assumption is unlikely to bias the results of this paper, because (i) in theoretical equilibrium, all operators have the same pricing schedule, (ii) in a counterfactual exercise of section 8, optimal prices are typically higher than currently existing ones.

Given this assumption, section 3 shows that the operator's optimal price is also linear in x and takes the form $p_{ij}(x) = p^a_{ij} + p'_i x$ for some p^a_{ij}, p'_i. Then, $z_{ij}(p_{ij}(x)) = \frac{p^a_{ij} - \tilde{p}^a_{ij} + (p'_i + w)x}{\tilde{p}'_i + w}$. Given linear price function, the mean walking distance (also equal to mean reservation time) before an $i \to j$ trip is equal to[3]

$$\bar{x}_{ij} = \frac{\int_{x=0}^{\infty} x D_{ij}(x, p_{ij}(x)) \mathrm{d}x}{\int_{x=0}^{\infty} D_{ij}(x, p_{ij}(x)) \mathrm{d}x} = \frac{\theta h_{ij}}{p'_i + w + 2\theta h_{ij}\left(\mu_i + \tilde{\mu}_{ij}\frac{p'_i + w}{\tilde{p}'_i + w}\right)}. \tag{3}$$

The mean walking distance increases with travel time h_{ij}, as people traveling further will search for an SV within a larger area around trip origin. The relationship is linear if there are no alternative options ($\mu_i = 0, \tilde{\mu}_{ij} = 0$) but is concave otherwise, as presence of other SVs imposes an upper bound on $\bar{x}_{ij}$. As one might expect, walking disutility w and reservation

[3]In (3) we assume distance x between a vacant vehicle and potential customer ranges from zero to infinity, while section 2.1 has assumed that both customer and vehicle are on the circle of finite size. While it is straightforward to impose a finite upper bound on x in (3) and thereafter, unbounded x proxies for the possibility of walk across two or more neighboring zones to reach a vacant SV. Such possibility is empirically plausible, if the value of travel v is high enough, while the density of vacant vehicles μ_i and price $p_{ij}(\cdot)$ are low enough.

cost p'_i reduce the walking range, while competitors' resevation cost $\tilde{p}'_i$ increases it.

Define the *departure rate* q_{ij} as the Poisson rate of vacant vehicle booking in zone i for a trip to j:

$$q_{ij} \equiv 2\lambda_{ij} \int_{x=0}^{\infty} D_{ij}(x)\mathrm{d}x = 2\lambda_{ij} \exp\left(-\frac{p^a_{ij}}{\theta h_{ij}} - 2\tilde{\mu}_{ij} \frac{p^a_{ij} - \tilde{p}^a_{ij}}{\tilde{p}'_i + w} \right) \bar{x}_{ij}. \tag{4}$$

2.5. Operator profit: formal definition

Define by the vehicle *cycle* the time between (i) the moment it becomes vacant in some zone and (ii) the moment it is dropped off by a customer in another zone. Each cycle then includes three stages: (i) vacancy, (ii) reservation while walking to the vehicle, and (iii) trip to the destination.

The mean reservation time $\bar{x}_{ij}$ in (3) can be viewed as function of p'_i, while the departure rate q_{ij} in (4) – as function of p^a_{ij} and p'_i. Denote by p^a_i the vector of $p^a_{ij}, \forall j \neq i$, and by $q_i(p^a_i, p'_i) = \sum_{j \neq i} q_{ij}(p^a_{ij}, p'_i)$ the overall departure rate out of zone i to any destination, so that $\frac{1}{q_i}$ is the expected duration of vacancy at i.

Denote by $s_{ij}(p^a_i, p'_i) \equiv \frac{q_{ij}(p^a_{ij}, p'_i)}{q_i(p^a_i, p'_i)}$ the probability that a vehicle cycle beginning at i will end at j. Denote by τ_i the expected duration of a cycle originating at i, $\tau_i(p^a_i, p'_i) = \frac{1}{q_i(p^a_i, p'_i)} + \sum_{j \neq i} s_{ij}(p^a_i, p'_i)(\bar{x}_{ij}(p'_i) + h_{ij})$. Denote by Ψ_i the expected gross profit per cycle originating at i,

$$\Psi_i(p^a_i, p'_i) = -\frac{g_i}{q_i(p^a_i, p'_i)} + (p'_i - g_i) \sum_{j \neq i} s_{ij}(p^a_i, p'_i)\bar{x}_{ij}(p'_i) + \sum_{j \neq i} s_{ij}(p^a_{ij} - ch_{ij}). \tag{5}$$

Finally, denote by $\mathbf{p}^a$ and $\mathbf{p}'$ the vectors of all p^a_{ij} and p'_i, respectively. Then, the vector $\{f_1, \ldots, f_{|\mathcal{I}|}\}$ of steady-state distribution of cycle origins satisfies

$$\sum_{j \neq i} s_{ji}(p^a_j, p'_j) f_j(\mathbf{p}^a, \mathbf{p}') = f_i(\mathbf{p}^a, \mathbf{p}'), \forall i. \tag{6}$$

Given the above notation, the expected gross profit flow ϕ is the maximal ratio of unconditional expected gross profit per cycle to the unconditional expected cycle duration:

$$\phi = \max_{\mathbf{p}^a, \mathbf{p}'} \frac{\sum_i f_i(\mathbf{p}^a, \mathbf{p}')\Psi_i(p_i^a, p_i')}{\sum_i f_i(\mathbf{p}^a, \mathbf{p}')\tau_i(p_i^a, p_i')}. \tag{7}$$

Maximization of (7) is equivalent to maximization of the expected gross profit *net* of foregone opportunities during the cycle:

$$\max_{\mathbf{p}^a, \mathbf{p}'} \sum_i f_i(\mathbf{p}^a, \mathbf{p}')(\Psi_i(p_i^a, p_i') - \phi\tau_i(p_i^a, p_i')). \tag{8}$$

Under the optimal (maximal) value of ϕ, the maximal value of (8) is zero.

3. Analysis

3.1. Symmetric model

To develop intuition of SV market equilibrium properties unrelated to spatial inequality, we first consider the simplest possible spatial structure with only two identical zones with symmetric travel demand between them. Due to symmetry, all location subscripts are dropped throughout this section. Because for any origin there is only one possible destination, we have $s_{ij} = 1$; symmetry of the two zones also implies $f_i = \frac{1}{2}$, hence the two components of the sum in (8) are identical.

3.1.1. Optimal price

The problem (8) simplifies to finding three scalars p^a (rental price except the reservation fees), p' (reservation fee per minute), and ϕ, that solve

$$\max_{p^a, p'} p^a + p'\bar{x}(p') - ch - g\left(\bar{x}(p') + \frac{1}{q(p^a, p')}\right) - \phi\left(h + \bar{x}(p') + \frac{1}{q(p^a, p')}\right), \tag{9}$$

subject to constraint that (9) is zero. Instead of immersing into formal algebra, I offer a more intuitive explanation of optimal solution. Consider a vacant SV parked at a representative location. A customer who appeared at distance x away from the SV will book it with probability $D(x, p(x))$. The operator's gross profit, net of foregone opportunities, is $p(x) - gx - ch - \phi(x + h)$.

The probability that a customer appears within time period dt and within space interval $[x, x+\mathrm{d}x]$ is $2\lambda \mathrm{d}t\mathrm{d}x$; the coefficient 2 here is because the customer may appear on both sides of the vehicle.

If the vehicle was not reserved during the time interval dt, the operator incurs a parking cost $g\mathrm{d}t$.

Given all of the above, gross profit is an outcome of optimization of the following objective:

$$\phi \mathrm{d}t = \max_{p(\cdot)} \mathrm{d}t 2\lambda \int_0^\infty D(x, p(x)) \left[p(x) - gx - ch - \phi(x + h)\right] \mathrm{d}x - \left[1 - \mathrm{d}t 2\lambda \int_0^\infty D(x, p(x)) \mathrm{d}x\right] g\mathrm{d}t. \tag{10}$$

From (10), the optimal price for a particular value of x can be found independently from other values of x, by maximizing the first integrand. The first-order condition reads

$$\frac{\mathrm{d}D(x, p)}{\mathrm{d}p} \left[p - ch - \phi(h + x) - gx\right] + D(x, p) = 0. \tag{11}$$

From (2), we have that $\frac{\mathrm{d}D(x,p)}{\mathrm{d}p} = -\frac{D(x,p)}{mh}$, where

$$m = \frac{\theta(\tilde{p}' + w)}{\tilde{p}' + w + 2\theta h \tilde{\mu}}. \tag{12}$$

Then, the optimal price $p(x)$ can be found from (11) as

$$p(x) = (m + c + \phi)h + (g + \phi)x. \tag{13}$$

Thus, the cost of the trip *per se* is $p^a = (m + c + \phi)h$ while the per-minute reservation rate is

$p' = g + \phi$. The value of p' implies the customer should pay for parking and for the foregone operator's profit while she is walking to the vehicle. The per-minute price for the trip *per se*, $\frac{p^a}{h}$, includes movement costs, opportunity costs, and the per-minute markup m.

How does the optimal trip price p^a depend on the trip duration h? From (12), the per-minute markup m is

- constant and equal to θ when there is no competition with other operators $\tilde{\mu} = 0$,
- decreasing with h from θ to zero when competition exists, $\tilde{\mu} > 0$.

Thus, the customer always pays $c+\phi$, i.e. the operating cost plus the foregone gross profit per minute; in addition, she should be charged a markup up to θ per minute, with highest markup corresponding to (i) a monopolized market or (ii) short trips. The markup increasing with monopolization is a standard result in the industrial organization literature. The markup decreasing with trip duration is because longer trip duration makes customers search for vacant vehicles in larger area, and to walk longer distances towards a cheap option, essentially increasing the price elasticity of demand.

3.1.2. Free entry and market equilibrium

This section characterizes the free-entry equilibrium density of vacant vehicles and factors affecting it. In equilibrium, all operators charge identical optimal rates for reserved vehicles $p' = \tilde{p}' = g + \phi$. The trip cost is common to all operators, too, $p^a = \tilde{p}^a$, meaning that $z = x$ in (1). The optimal markup (12) is then $m(\tilde{\mu}) = \frac{\theta(g+\phi+w)}{g+\phi+w+2\theta h\tilde{\mu}}$. Furthermore, assume all operators are small so $\mu = 0$; then the departure rate (4) is simplified to $q(\lambda, \tilde{\mu}) = 2\lambda \exp\left(-\frac{m(\tilde{\mu})+c+\phi}{\theta}\right) \frac{m(\tilde{\mu})h}{g+\phi+w}$.

Then, dropping the second-order term in (10), and dividing both sides by dt, we obtain

$$\phi + g = q(\lambda, \tilde{\mu})m(\tilde{\mu})h. \qquad (14)$$

In the above equation, the left-hand side is the expected per-minute cost of a vacant vehicle, equal to the sum of foregone gross profit and parking cost. The right-hand side is the expected revenue flow per vacant vehicle per minute, equal to the product of the departure rate, per-minute price markup, and trip duration.

In free-entry equilibrium, all operators are breaking even, meaning that the equilibrium gross profit ϕ is exogenous and equal to per-minute fixed vehicle cost. Then, the equation (14) is a condition on equilibrium density of vacant vehicles $\tilde{\mu}$, which enters the right-hand side of (14) via $m(\tilde{\mu})$ and $q(\lambda, \tilde{\mu})$. We can study the effects of exogenous parameters on such density. For example, because both $m(\tilde{\mu})$ and $q(\lambda, \tilde{\mu})$ are decreasing with $\tilde{\mu}$, while q is proportional to λ, we can conclude that a higher demand density λ is increasing $\tilde{\mu}$. The relationship however is not linear. For λ smaller than $\lambda_0 = \frac{1}{2}\frac{(g+\phi)(g+\phi+w)}{(\theta h)^2}\exp\left(1+\frac{c+\phi}{\theta}\right)$ such that $\phi + g \equiv q(\lambda_0, 0)m(0)h$, shared vehicle service is not sustainable and the equilibrium vehicle density is zero. This is because, even in the absence of competition, demand for a vacant vehicle is limited by the walking costs. But as the demand density λ rises to infinity, vehicle density is asymptotically a square root of demand density, $\tilde{\mu} \approx \left(\frac{\lambda}{2}\exp\left(-\frac{c+\phi}{\theta}\right)(g+\phi+w)\right)^{\frac{1}{2}}$. The relationship is concave because higher vehicle density reduces both price markup and the customer walking range, thus affects expected revenue more strongly than demand density does.

Besides the density of vacant vehicles $\tilde{\mu}$, it is useful to calculate the density of vehicles reserved by walking customers, as well as the number of vehicles in transit. For that purpose, observe that the c.d.f. of the walking time x from customer to the nearest vehicle is $1 - \exp(-2\tilde{\mu}x)$, hence the p.d.f. is $2\tilde{\mu}\exp(-2\tilde{\mu}x)$. The probability that a customer facing a walk of x minutes will accept it is $\exp(-\frac{p(x)+wx}{\theta h})$, hence the expected walking time of a typical customer, including those who rejected the opportunity and did not walk, is

$$\int_{x=0}^{\infty} x \exp\left(-\frac{p(x)+wx}{\theta h}\right) 2\tilde{\mu}\exp(-2\tilde{\mu}x)\mathrm{d}x$$
$$= 2\tilde{\mu}\exp\left(-\frac{m+c+\phi}{\theta}\right)\left(\frac{\theta h}{\phi+g+w+2\theta h\tilde{\mu}}\right)^2. \quad (15)$$

The density of reserved vehicles must then be equal to the above expected walking time, multiplied by density of new customers λ. By comparing the resultant expression against the equilibrium condition (14), we conclude that the equilibrium density of reserved vehicles is $\frac{\phi+g}{\phi+g+w}\tilde{\mu}$, i.e. is a constant fraction of the density of vacant vehicles, and therefore is also asymptotically proportional to $\lambda^{\frac{1}{2}}$.

To find the equilibrium number of vehicles in transit, per unit of zone space, we replace the waking time x in (15) by the transit time h and multiply by the density of customers λ to arrive at $2\lambda\tilde{\mu}\exp\left(-\frac{m+c+\phi}{\theta}\right)\frac{\theta h^2}{\phi+g+w+2\theta h\tilde{\mu}}$. Asymptotically, as both λ and $\tilde{\mu}$ grow large, this quantity approaches $\lambda h\exp\left(-1-\frac{(c+\phi)}{\theta}\right)$, i.e. is asymptotically proportional to the demand density λ. We may conclude that shared vehicles will sit more time vacant in low-density municipalities, which justifies the lower bound on demand density for the SV service to become viable. Figure 2 visualizes these findings.

3.2. General model

3.2.1. Optimal price revisited

We now return to the general model with heterogenous demand densities and heterogenous trip durations. The main point of this section is to argue that the result of section 3.1.2, that the SV service is not viable in low-density areas, may be reversed if such an area is proximate to a high-density area and if the pricing method is optimal. Today, SV operators around the world avoid serving low-density suburban areas, so the analysis of this paper has a potential to expand the SV service areas, and to make suburban trips feasible.

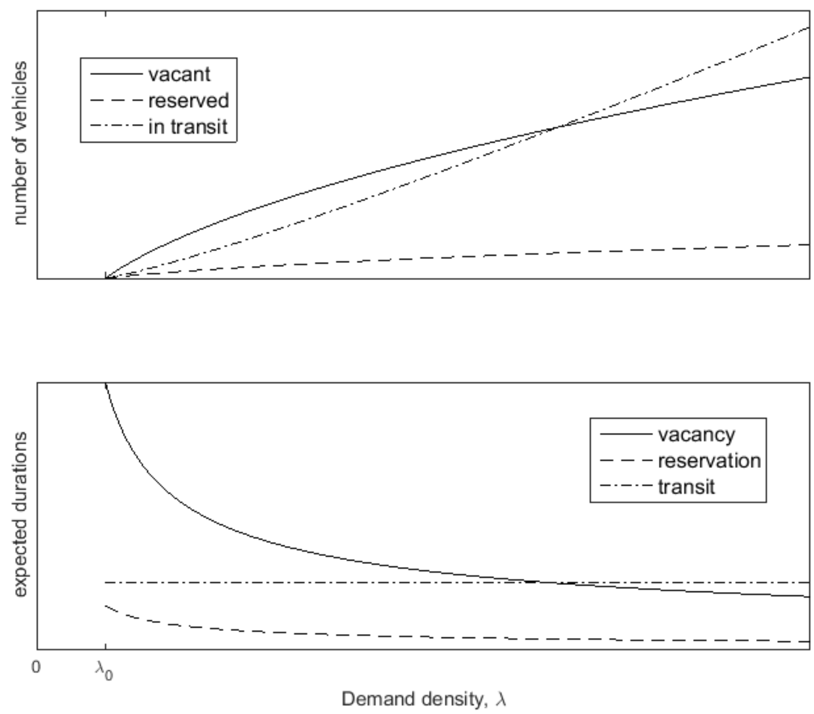

Figure 2: Number of vehicles and expected durations by phase

When zones are heterogenous, presence of a vacant vehicle in different zones may be of different value to the operator. For example, presence of a vehicle in a low-demand-density zone may reduce the operator's profit due to longer expectation of the next customer. To quantify such heterogeneity, denote by R_i the value of a vacant vehicle being in zone i, relative to the value of being in the "numeraire" zone 1 with $R_1 = 0$. Intuitively, R_i is the expected profit gain from the fact that the vehicle is in i rather than in zone 1. Alternatively, R_i can be interpreted as the expected cumulative profit, on top of "normal" profit flow ϕ, for a vacant vehicle in zone i, until the moment it ends up in the "numeraire" zone 1.We will nickname R_i as the *score* of zone i.

To formulate R_i mathematically, we generalize (10) as follows:

$$R_i = \lim_{\mathrm{d}t \to 0} \mathrm{d}t \max_{p_{ij}(\cdot),\forall j} \sum_{j \neq i} 2\lambda_{ij} \int_{x=0}^{\infty} D_{ij}(x, p_{ij}(x)) \left[p_{ij}(x) - ch_{ij} - \phi(h_{ij} + x) - g_i x + R_j\right] \mathrm{d}x + \left[1 - \mathrm{d}t \sum_{j \neq i} 2\lambda_{ij} \int_{x=0}^{\infty} D_{ij}(x, p_{ij}(x)) \mathrm{d}x\right] \left[-(g_i + \phi)\mathrm{d}t + R_i\right]. \quad (16)$$

The first component on the right-hand side of (16) is the sum across all possible destinations j of the chance of departing to this destination, times the expected gross profit from such a trip, net of foregone opportunities, plus the value R_j of ending up there. The second component on the right-hand side of (16) is the chance of not finding a customer during time interval dt, times foregone opportunities plus the value R_i of remaining in the same place.

Replicating the steps following (10) and denoting (cf.(12)) $m_{ij} \equiv \frac{(\tilde{p}'_i + w)\theta}{\tilde{p}'_i + w + 2\tilde{\mu}_{ij}\theta h_{ij}}$, we find the price function $p_{ij}(\cdot)$ maximizing the right-hand side of (16), as follows (cf.(13)):

$$p_{ij}(x) = (m_{ij} + c + \phi)h_{ij} + (g_i + \phi)x + R_i - R_j. \quad (17)$$

Intuitively, the price consists of three elements. The first two elements, payments for the trip *per se* and for reservation time, are analogous to those of section 3.1.1. The novel element is relocation fee $R_i - R_j$, i.e. a lump-sum fee (bonus) for decreasing (improving) the location score of the vehicle.

In the notation of section 2.4, we have that $p^a_{ij} = (m_{ij} + c + \phi)h_{ij} + R_i - R_j$ while $p'_{ij} = g_i + \phi$. Then, the departure rate (4) under optimal pricing is

$$q_{ij} = 2\lambda_{ij} \exp\left(-1 - \frac{c+\phi}{m_{ij}} + \frac{R_j - R_i}{m_{ij}h_{ij}} + \frac{2\tilde{\mu}_{ij}\tilde{p}^a_{ij}}{\tilde{p}'_i + w}\right)\bar{x}_{ij}. \tag{18}$$

By plugging (17) back into (16), dropping the second-order terms, and rearranging, we end up with the following system of equations (cf.(14,18)):

$$g_i + \phi = \sum_{j \neq i} q_{ij} m_{ij} h_{ij}, \forall i. \tag{19}$$

The interpretation of all elements in (19) is similar to that of (14). This is a system of $|\mathcal{I}|$ equations with an equal number of unknowns: gross profit flow ϕ and $|\mathcal{I}| - 1$ zone scores $R_i, i = 2 \ldots |\mathcal{I}|$.

3.2.2. Equilibrium properties

Because both m_{ij} and h_{ij} in (19) are bounded from above, (18) implies that a low demand density $\lambda_{ij}, \forall j \neq i$ in some origin zone i should be offset by a reduced zone score R_i. The relationship here is logarithmic, so when demand density approaches zero, the zone score approaches negative infinity. At the same time, R_i has a finite upper bound because (i) the demand density is finite, (ii) $R_1 = 0$ and the difference $R_i - R_1$ must be finite due to connectedness of zones 1 and i.

A lower zone score, in turn, means that the trip cost is increased for the customer taking the vehicle to that zone, and is reduced by the same amount for the next customer who will take the vehicle out of the zone. Intuitively, a price discount for outbound customers helps to boost departures from low-demand zones; the discount is funded by the previous customer who left the vehicle in such zone.

Likewise, a higher parking rate g_i reduces the zone score R_i. For example, if an SV operator expands its dropoff area to a private garage with parking rates higher than those of nearby municipal parking, the garage should be assigned a lower score. This means that a customer who drops off a vehicle at the garage will be charged an extra lump-sum fee, while the next customer who takes the vehicle out of the garage will earn a lump-sum bonus of the same amount; the bonus accelerates the departure out of the garage. From the SV operator's perspective, higher monetary cost of vacancy at the garage is offset by a shorter vacancy period (thus less foregone opportunities); gains and losses should exactly offset each other if the zone scores are right.

The optimal price (17) implies that the trip cost may depend on the origin location not only via the zone score R_i, but also via the trip-specific per-minute markup m_{ij}, which in turn decreases with density $\tilde{\mu}_{ij}$ of competing vehicles that can be rented for the same trip. This creates an opportunity for *trip arbitrage*: if markup m_{ij} is high due to low competition $\tilde{\mu}_{ij}$, customers may have an incentive to split the trip $i \to j$ into two trips, $i \to k$ and $k \to j$ with zero stopover time, provided that either m_{ik} or m_{kj} are sufficiently lower than m_{ij} and the total trip duration $h_{ik} + h_{kj}$ is not much higher than h_{ij}. For example, a customer traveling from a city center i to a distant suburb j may split the trip into two legs with a stopover at the city boundary k, if high competition in SV service within the city dampens the markup m_{ik}. To eliminate incentives for trip arbitrage, adjustments to the markup m_{ij} can be devised; generally, such adjustments should reduce the dependence of m_{ij} on trip-specific competition $\tilde{\mu}_{ij}$.

3.3. Computation of optimal policy

Section 3.2.1 has determined that the profit-maximizing price takes the form of (17). In that price, the endogenous unknown parameters are ϕ and $\mathbf{R} = \{R_2, \ldots, R_{|\mathcal{I}|}\}$. Theoretically, these unknowns can be found from (19), which defines a system of $|\mathcal{I}|$ nonlinear equations. However, as the number of zones $|\mathcal{I}|$ is measured in the thousands in the empirical application below, the prospect of finding a solution to this system is obscure.

The current section specifies a method to find optimal policy parameters via a more robust approach. I view all price variables in section 2.5 as functions of ϕ and $\mathbf{R}$ given by (17); then, optimal vector $\mathbf{R}$ should maximize (8), while equilibrium ϕ makes it equal to zero.

It is convenient to define by $\Phi_i(\phi, \mathbf{R})$ the part of Ψ_i that does not include fees for zone score change (cf.5): $\Phi_i(\phi, \mathbf{R}) = -\frac{g_i}{q_i(\phi, \mathbf{R})} + \phi \sum_{j \neq i} s_{ij}(\phi, \mathbf{R}) \bar{x}_{ij}(\phi) + \sum_{j \neq i} s_{ij}(m_{ij} + \phi) h_{ij}$, so that $\Psi_i(\phi, \mathbf{R}) = \Phi_i(\phi, \mathbf{R}) + R_i - \sum_{j \neq i} s_{ij} R_j$. We have that

$$\sum_i f_i \Psi_i = \sum_i f_i \left[\Phi_i + R_i - \sum_{j \neq i} s_{ij} R_j \right] = \sum_i f_i \Phi_i + R_i \left[f_i - \sum_{j \neq i} s_{ji} f_j \right] \underbrace{=}_{(6)} \sum_i f_i \Phi_i.$$

Intuitively, zone score fees are equal to zero in expectation and thus do not affect the expected revenue directly; their only role is to regulate the rate of arrivals and departures at various locations. Given the above result, the maximization problem (8) is equivalent to

$$\max_{\mathbf{R}} \sum_i f_i(\phi, \mathbf{R}) (\Phi_i(\phi, \mathbf{R}) - \phi \tau_i(\phi, \mathbf{R})). \tag{20}$$

In that problem, each element of the sum

$$\Phi_i - \phi \tau_i = -\frac{g_i + \phi}{q_i} + \sum_{j \neq i} \frac{q_{ij}}{q_i} m_{ij} h_{ij}$$

is equal to zero at the optimal point, according to (19). But then, the marginal effect of a changing f_i is zero, too, at the optimal point. The problem that ϕ in (20) is unknown ex-ante can be addressed in two ways: (i) using the fact that (20) is zero at optimal point as optimization constraint; (ii) start with low value of ϕ, maximize (20) to find optimal $\mathbf{R}$, calculate new (higher) ϕ from (7), repeat the loop until convergence.

4. An empirical application: overview

The rest of the paper fits the above theoretical model to the carsharing market of Moscow, Russia, and conducts a counterfactual experiment that models a hypothetical new operator that utilizes the optimal location-specific pricing derived in section 3.2.1.

According to truesharing.ru, Moscow was served by 9 free-floating carsharing operators as of late 2019; additional 3 operators specialize in certain suburban areas of the city. The fleet size has doubled in 2019, reaching approximately 30000 vehicles by the end of the year. Of these, over 95% are operated by the "Big Four" operators, Yandex Drive (about 50% of the market), Delimobil (30%), BelkaCar and YouDrive. Vehicle types, rates, and home areas are quite similar across operators. The primary tariff is a location-invariant per-minute rate, which may depend on the time of the day; daily and sometimes 3-hour tariffs are also available. Operators also charge a much lower *parking rate* while the vehicle is reserved but not moving, and universally offer several minutes of free reserved time to walk to the vehicle. The home areas, where the vehicles must be returned, of all operators include nearly all territory inside the MKAD, the loop highway around the city with 15-20 km radius, as well as certain hand-picked high-density areas outside of MKAD. The outside-MKAD home area is quite limited: approximately 280 km^2 for Delimobil, 180 km^2 for Yandex Drive, and only 95 km^2 for BelkaCar,[4] i.e. a small fraction of the 30000 km^2 Moscow region, or even of 900 km^2 of the inside-MKAD territory. Figure 3 illustrates the home area of Delimobil (i.e. the largest among existing operators) within the studied geographic area.

The rest of the paper is structured as follows. Section 5 describes the data used in the

[4]Author's own estimate

Figure 3: Studied geographic area. Home area of *Delimobil* highlighted in red. Sources: Bing maps, Delimobil.

empirical study, section 6 calibrates some model parameters while section 7 estimates the remaining parameters. Section 8 introduces a counterfactual SV operator that uses optimal pricing and competes with existing operators. Section 9 concludes.

5. Data

5.1. Shared car data

The empirical model is based on real-time data on vacant shared vehicles around Russia published at an aggregator website carsharing.gde-luchshe.ru. The website collects data from operators representing about 40% of the Moscow area SV market, and publishes it in HTML format. Among the Big Four operators mentioned in section 4, data from Delimobil and YouDrive is available. The data includes: vehicle location, make, operator, fuel left. Importantly, a unique vehicle ID is also provided, which allows us to track vehicles over time. Kortum et al. (2016) and Wielinski et al. (2018) use a similar data collection method to analyze SV trips in Western Europe and in North America.

The data used in this paper was downloaded with 5-minute intervals from 10:43am on December 17, 2019, until 8:48am on December 24, 2019, with occasional lapses due to website unavailability. The total number of time observations is 1976. The geographic coverage, *the Moscow area* henceforth, includes 30 arc-minutes of latitude (i.e. 55.5 km) North and South from the Red Square in Moscow, and one degree of longitude (62.1 km) East and West from the same location. Specifically, the studied area is confined between $[55°15'\text{N}, 56°15'\text{N}]$ of latitude and $[36°37'\text{E}, 38°37'\text{E}]$ of longitude, and is illustrated on figure 3. In total, 15.5 million vehicle availability events were recorded within the above time frame and geographic area. A tiny fraction of these observations were dropped due to lack of credibility, e.g. when a vehicle longitude was exactly 37°E and corresponded to a wilderness area without roads.

A total of 10457 unique vehicles were observed, belonging to 8 operators. Of these vehicles, 8727 (83.5%) belong to Delimobil, 1276 (12.2%) to YouDrive, and another 454

Vehicle make	Count	Percent
VW Polo	6058	57.93
Hyundai Solaris	1150	11.00
Nissan Qashqai	763	7.30
Smart Fortwo	615	5.88
KIA Rio	439	4.20
BMW 3	383	3.66
Renault Kaptur	247	2.36
other	802	7.67

Table 1: Distribution of vehicle types

Percentile	10	25	50	75	90	99
Rental duration, min	15	25	40	65	100	245

Table 2: Percentiles of rental duration distribution

(4.3%) to another 6 operators. The distribution of vehicle types is provided in table 1; the lion's share of vehicles is economy class such as VW Polo and Hyundai Solaris.

Because the actual revenue trips are not directly observed, they have to be inferred from the data. For a given vehicle, by a *movement event* I define a combination of start and end locations and times such that (i) the vehicle was not available between start and end times and (ii) either latitude or longitude of the vehicle have changed by more than 100 meters. A total of 397211 such movement events were identified. For every such event, define the *movement speed* as the ratio of (i) the geodesic distance between start and end locations and (ii) difference between end and start times. I discard movement events with (i) speed exceeding 60kmh (0.4% of all observations), as these are often errors in location recording, and (ii) speed below 2kmh (17.4% of observations), which are likely to include non-revenue trips, or rental trips with long stopovers, or rental round-trips. The remaining movement events are labeled as (one-way) *rental events*; the duration of each rental event includes the walking time x and the time of the trip per se, h. The mean duration of a rental event is 52 minutes; some distribution percentiles are shown in table 2.

Because data was recorded with 5-minute intervals, all rental durations are rounded upward. Also, two or more consecutive rental events of the same vehicle could have been recorded as one, if the vacancy period between the trips fell within a 5-minute interval between data recordings.

Not surprisingly, there is a clear daily cycle in travel demand. The peak demand is observed during rush hours of 6-8 am and 7-9pm, with 3000+ rental events per hour initiated during this time. The lowest demand is between 3am and 4am, with less than 400 rental events per hour initiated. While time-varying demand can make optimal pricing to vary over time, as well, this paper considers only time-invariant prices for transparency of other results, leaving the time dimension for future research.

5.2. Night light data

To predict travel demand to/from suburban areas currently not served by SV operators, we associate such demand econometrically with the data on night light radiance, commonly used in economics to predict local economic activity. I use the standard source of such data, published by the Earth Observations Group at NOAA/NCEI. The data is provided monthly for every rectangle of the Earth surface measuring 15 arc-seconds of latitude (about 463m) and 15-arc seconds of longitude (259m at the latitude of Moscow). The zones defined in section 2 are set equal to these nightlight rectangles in the empirical analysis below. Thus, the Moscow area spans 240 zones North-South and 480 zones East-West, each zone occupying 12 ha of land.

For each zone, the nightlight data available is a value of radiance measured in 10^9 nanoWatt/cm^2/sr, *units* henceforth. To eliminate short-term lights caused by construction or social events, I take the minimum of two 2019 months, November (latest available at the time of download) and March. Figure 4 illustrates the resultant nightlight data, overlapped with Delimobil home area. A high correlation between SV home area and nightlight radiance is clearly visible, which confirms strong correlation between night lights and travel demand.

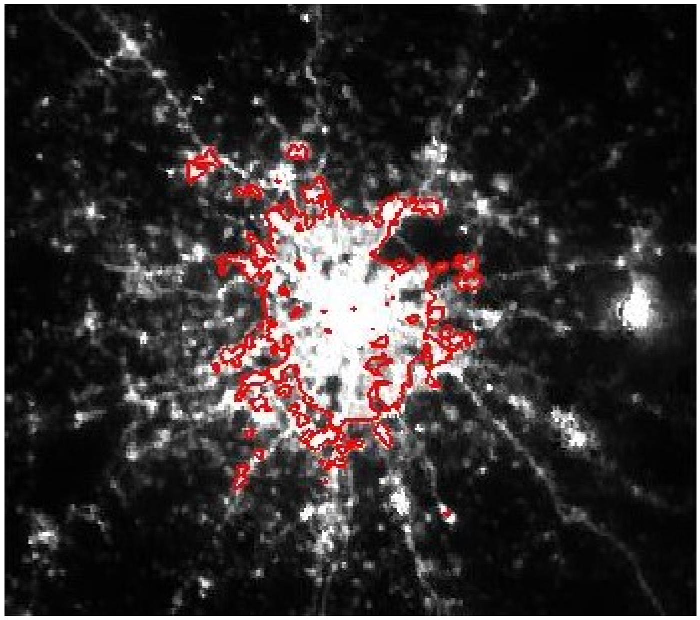

Figure 4: Nightlights within studied geographic area. Home area of *Delimobil* highlighted in red. Sources: NOAA/NCEI, Delimobil.

Zones with radiance below 10 units are typically located in the wilderness areas, such as forests or bodies of water, with unlikely potential for travel demand. In the data, 72.5% of all zones have radiance below this threshold and are excluded from the analysis. In contrast, the median radiance inside the MKAD is 47.7; the radiance at the Red Square, the top tourist attraction, is about 250.

There are three areas outside of MKAD with extremely high radiance of up to 12700 units, which correspond to large greenhouse facilities. In the analysis, we drop all zones currently served by SV operators with radiance exceeding 500 units (these are residential districts adjacent to greenhouses), as well as all other zones with radiance over 100 units (these zones would have been included by current operators if high travel demand indeed existed).

5.3. Home areas of SV operators

As mentioned above, most destinations outside of MKAD are currently not served by SV operators. To correctly assess the model parameters, it is therefore essential to know which zones in our model are included into the home area of which operator. This paper uses the data on the boundaries of home areas of the Big four operators in Moscow. Even though we do not have trip data for two of these operators, Yandex Drive and BelkaCar, information on their home areas is used to assess the density $\tilde{\mu}$ of available vehicles by "other operators."

Moscow SV operators do not publish their home areas in a machine-readable format; the data for each operator is available only in the image format via the operator's mobile app. To digitize the data, screenshots from the apps were made, geolocated, and processed.

6. Calibrated model parameters

Shared vehicle operators are exempt from paying parking fees in the city of Moscow. Thus, we assume $g = 0$ throughout the rest of the paper.

6.1. Density of competing vehicles

The theoretical model of section 2 studies a space with a single dimension, while the data on vehicle location is obviously two-dimensional. Thus, we need to specify a method to map the empirical density $\tilde{M}$ of competing vehicles, measured in the number of vehicles per zone, to the theoretical density $\tilde{\mu}$, measured in the number of vehicles per minute of linear walking distance. I create such mapping by matching the expected walking time to the nearest vacant SV predicted by the linear and by two-dimensional models.

In the linear model, the probability that the nearest vehicle is less than x minutes away is $1 - \exp(-2\tilde{\mu}x)$, and the expected walking time to the nearest vehicle is $\frac{1}{2\tilde{\mu}}$ minutes. In the two-dimensional model, assuming walking speed of 60m/min, a customer walking x minutes can cover area $3600\pi x^2$ sq.m. With $\tilde{M}$ vacant competing vehicles per zone measuring

$463 \times 259 = 119917$ sq.m, this area will have $0.03\pi\tilde{M}x^2$ vehicles on average. The probability of finding at least one vehicle within this area is $1 - \exp(-.03\pi\tilde{M}x^2)$, and the expected walking time to the nearest vehicle is $\int_{x=0}^{\infty} x\mathrm{d}(1-\exp(-.03\pi\tilde{M}x^2)) = \frac{2.8868}{\tilde{M}^{\frac{1}{2}}}$. By equating the expected walking times in the two models, we end up with $\tilde{\mu} = 0.1732\tilde{M}^{\frac{1}{2}}$. As mentioned earlier, calculation of trip-specific $\tilde{\mu}_{ij}$ involves vacant vehicles in zone i by operators that also serve zone j.

While the density of available vehicles by most operators is directly observed and included into the calculation of $\tilde{M}$, there is no data from two major operators, Yandex Drive and BelkaCar. The total market share of all observed operators is estimated at about 40%, while Yandex Drive is known to have about 50%. Thus, we estimate the density of Yandex Drive vehicles in every zone i within its home area at $1.25M_i^e$, where M_i^e is the average number of vehicles by all observed operators in zone i. For BelkaCar, its approximate market share is 10%, hence the estimated number of vehicles in a zone i within BelkaCar's home area is $0.25M_i^e$. Other unobserved operators are very small and can be ignored.

6.2. Trip durations

Because observed rental durations include not only trip duration h but also walking time x, the trip durations are not known precisely. I will assume that h_{ij} is an increasing function of the geodesic distance between origin and destination locations, d_{ij}, which is measured with high degree of accuracy. The relationship between h_{ij} and d_{ij} is inferred from rental events where the vehicle is deemed to be moving almost all the time. Specifically, we focus on rental events that satisfy the following two criteria.

Density The theory of this paper predicts that the walking time x is inversely related to the density $\tilde{\mu}$ of available vehicles, and approaches zero when $\tilde{\mu} \to \infty$. Thus, trips originating at zones with high vehicle density are likely to consist almost entirely of vehicle movement time h. We will focus on trips with 10+ vehicles available in the origin zone at the beginning of the rental period.

Fuel consumption Because vehicles burn fuel only while in transit, data on fuel available allows to assess how long a vehicle has been moving. I focus on the analysis of VW Polo, the most common vehicle in the dataset. The vehicle specifications indicate fuel consumption of 7.9 l per 100 km in the "urban driving cycle". This measurement is done following a standard international protocol of various vehicle movements. I assume that, because driving conditions in Moscow may differ from that protocol, fuel consumption *per kilometer* may differ, but fuel consumption *per hour* is the same. Specifications of the urban driving cycle imply an average speed of 18.35 km/h, implying that VW Polo burns 1.45 l (2.63 % of fuel tank) per hour. Assuming this is the lower bound on fuel consumption,[5] we consider only events of VW Polo rentals with fuel consumption above this bound.

A total of 5027 rental events satisfy the above criteria. Among these, there is a correlation of 72% between the trip distance and the movement speed. This finding can be explained by the fact that high-speed roads are more likely to be used for longer-distance trips. To account for the fact, we calibrate the trip duration as follows:

$$h_{ij} = h(d_{ij}) = d_{ij} \frac{(12 + \alpha_1 d_{ij}^{\alpha_2})}{1 + d_{ij}^{\alpha_2}}, \tag{21}$$

with $\alpha_1, \alpha_2 > 0$, so the ratio $\frac{h_{ij}}{d_{ij}}$ drops from 12 min/km (i.e. pedestrian speed) when $d_{ij} = 0$ down to α_1 when $d_{ij} \to \infty$. The values of $\{\alpha_1, \alpha_2\}$ are found by fitting the log of (21) to the above subsample of 5027 rental events assuming their durations consist entirely of travel time h. I find $\{\alpha_1, \alpha_2\}$ to be equal to $\{2.57, 0.636\}$, so a 5-km-geodesic-distance trip takes on average 25 min, while a 25-km trip takes 91 min. Given function (21) and observed

[5] Compared to other driving modes, the "urban driving cycle" results in the highest fuel consumption per kilometer, but lowest per hour.

distribution of trip distances, the average trip duration in the entire sample of rental events is $Eh = 37.1$ min, or 70.3% of average rental duration $E(h + x)$.

6.3. Demand density

The demand density λ_{ij} is assumed to follow the gravity law $\lambda_{ij} = A\lambda_i\lambda_j h_{ij}^{-\gamma}$, where λ_i is nightlight radiance in zone i, while parameters A and γ are estimated in section 7.

6.4. Trip cost

The per-minute trip cost c is calibrated from 24-hour rental rates offered by Delimobil. That contract includes 70 free kilometers of travel, with a charge of 8 RUB per every additional kilometer. Assuming the latter number is the marginal cost of travel under the highest empirically possible urban speed of 40kmh, we assess the per-minute trip cost at $c = 8 \times 40/60 = 5.33$ RUB/min.

6.5. Mean value of travel

In the model of SV demand of section 2, the parameter θ is not identified. For example, an increase in θ, which proportionately increases all θh_{ij}, will result in the same departure rate q_{ij} if complemented by a proportionate increase in w and exponential increase in all λ_{ij} in (4).

This paper derives θ from the supply side of the SV market, from the following identifying assumption. Although theoretical analysis above shows that the existing pricing policy of flat per-minute rate and free walking time is not optimal, we will assume that operators behave optimally within the class of such policies and set the per-minute rate r to maximize their profit. To find optimal r within this class, we reiterate optimal policy calculation of section 3.1, maximizing the expected gross profit per minute.

If an operator charges the per-minute rate r while other operators charge $\tilde{r}$, we have that $p' = \tilde{p}' = 0$, $p^a = rh$, $\tilde{p}^a = \tilde{r}h$, hence (cf.(1)) $z = x + (r - \tilde{r})\frac{h}{w}$. Then, demand (2) can be rewritten as

$$D(x) = \exp\left(-\frac{r}{\theta} - \frac{w}{\theta h}x - 2\mu x - 2\tilde{\mu}\left(x + (r - \tilde{r})\frac{h}{w}\right)\right), \tag{22}$$

while the operator's profit per vehicle per time period dt is (cf.(10), recall $g = 0$)

$$\mathrm{d}t\lambda \int_{x=0}^{\infty} D(x)\left[(r-c)h - \phi(h+x)\right]\mathrm{d}x.$$

Profit maximization w.r.t. r yields (assuming $r = \tilde{r}$ in equilibrium)

$$r - c - \phi\left(1 + \frac{\theta}{w + 2(\mu + \tilde{\mu})\theta h}\right) = \frac{\theta w}{w + 2\theta\mu^a h}. \tag{23}$$

The ratio on the left-hand side of (23) is the ratio of mean walking time (cf.(3)) $\bar{x} = \frac{\theta h}{w + 2\theta h(\mu + \tilde{\mu})}$ to the trip duration h; this is because the opportunity cost of reservation time is included into the movement cost rather than charged separately. The entire left-hand side is the price markup (net of opportunity costs) per minute of trip duration.

To calibrate the per minute rate r, we use the price schedule of Delimobil that charges a flat per-minute rate depending on the time of the day and ranging within 7-8 RUB/min, except the morning rush hour with 5 RUB/min. Averaging the price across all rental events yields approximately $r = 6.85$ RUB/min (0.11 USD/min in December 2019). The gross profit ϕ is calibrated by assuming it is equal to the fixed cost of operation, which in turn consists mainly of vehicle purchase costs. Assuming new economy-class vehicles are bought for 800K RUB, funded via 3-year loans with 12% interest, and sold for 400K RUB after three years, the associated funding cost is 574 RUB per day. After adding other fixed costs such as insurance and taxes, I estimate the fixed cost to be about 700 RUB (11.1 USD in Dec.2019) per day. Assuming a typical vehicle is in commercial service (i.e. vacant or rented) for 20 hours per day, we infer $\phi = 700/20/60 = 0.583$ RUB/min. Section 6.2 also finds that an average trip duration is fraction $\frac{h}{h+Ex} = 0.703$ of of average rental duration, thus the price markup (the left-hand side of (23)) is equal to $6.85 - 5.33 - .583/.703 = 0.69$ RUB/min.

On the right-hand side of (23), the parameter $\tilde{\mu} \times h = 0.1732(\tilde{M})^{\frac{1}{2}} \times h$ is assumed to equal its empirical average across observed rental events, which is 8.19. Thus, the constraint (23) can be rewritten as

$$0.69 = \frac{\theta w}{w + 16.38\theta}, \tag{24}$$

which serves as a constraint on the optimization problem of section 7 that allows to identify θ.

7. Estimated model parameters

This section estimates the remaining unknowns w (walking cost) and A, γ (parameters of demand density λ_{ij}) of the theoretical model of section 2, fitting the model to the SV trip demand data outlined in section 5. To remove potential effects of the operator or vehicle type, we will focus on the economy-class vehicles by Delimobil, which include various versions of Hyundai Solaris, Kia Rio, Renault Kaptur and Sandero, and VW Polo. The vehicles meeting these criteria constitute 75.4% of all 10457 vehicles observed in the Moscow area. We will refer to these as the *sample vehicles*. Such sample of vehicles not only includes most observations, but is also most relevant for a study of a potential expansion of service to new suburban zones, because (i) Delimobil already serves more outside-MKAD areas than other operators, and (ii) suburban residents tend to have lower incomes and are thus more likely to target economy-class vehicles.

7.1. The econometric model

The unit of analysis is a zone within the Delimobil home area (8778 zones total) observed at a given moment of time. At every such zone i at every time moment t, measured in minutes, a number of sample vehicles is available. For every such vehicle, one of three events within the next five minutes (i.e. until moment $t + 5$) may occur:

- someone rents the vehicle for a trip to some other zone j;
- the vehicle remains vacant at the same location at $t+5$;
- the vehicle becomes unavailable for another reason, e.g. a maintenance trip, or it was reserved for some time but did not move.

The latter outcome is considered an exogenous shock to vehicle demand and is ignored in the analysis. Model parameters will be chosen to maximize the likelihood of the first two outcomes, conditional on the third outcome not happening.

The demand for a sample vehicle at time t in zone i is a location-specific version of (22), assuming $\tilde{r}=r$: $D_{ijt}(x)=\exp\left(-\frac{r}{\theta}-\frac{wx}{\theta h_{ij}}-2(\mu_{it}+\tilde{\mu}_{ijt})x\right)$, where $\mu_{it}+\tilde{\mu}_{ijt}$ is the density of other vacant vehicles in zone i at time t, belonging to Delimobil (μ_{it}) and to other operators serving destination j ($\tilde{\mu}_{ijt}$). The departure rate q_{ijt} is still defined by (4); the probability that a vehicle in zone i remains vacant after infinitesimal time dt is $1-\mathrm{d}t\sum_{j\neq i}q_{ijt}$. The probability of vacancy after five minutes is then

$$P_{i0t}=\exp\left(-5\sum_{j\neq i}q_{ijt}\right), \tag{25}$$

while the probability that the vehicle departs to a zone j is proportional to the instantaneous probability of the same event:

$$P_{ijt}=(1-P_{i0t})\frac{q_{ijt}}{\sum_{k\neq i}q_{ikt}}. \tag{26}$$

The theoretical outcome probabilities are fitted to empirically observed outcomes. The

Parameter	$\log(A)$	w	γ	θ
Value	-6.81	13.5	2.04	4.28
Standard deviation	0.769	1.23	0.189	2.03

Table 3: Estimated model parameters

loglikelihood function to be maximized is

$$\mathcal{L} = \sum_{i,t} n_{i0t} \ln(P_{i0t}) + \sum_{j \neq i} n_{ijt} \ln(P_{ijt})$$
$$= -5 \sum_{i,t} n_{i0t} \sum_{j \neq i} q_{ijt} + \sum_{i,t} \sum_{j \neq i} n_{ijt} \left[\ln(1 - \exp(-5 \sum_{k \neq i} q_{ikt})) - \ln(\sum_{k \neq i} q_{ikt}) \right] + \sum_{i,t} \sum_{j \neq i} n_{ijt} \ln q_{ijt}, \tag{27}$$

where n_{ijt} is the number of vehicles rented for a trip from i to j during time interval $[t, t+5]$, while n_{i0t} is the number of vehicles remaining vacant in zone i during the same time interval.

7.2. Estimation results

Because of large dimensionality of the problem (8778 origins, same number of destinations, 1974 time moments), some adjustments were made to make it computationally feasible. I used a subsample of time moments observed every 30 minutes, i.e. roughly 1/6 of all data. In addition, destinations were aggregated into clusters of 4×4 zones. The results of maximization of (27) over A, w, γ, θ, subject to (24), are outlined in table 3. The value of w implies that a typical customer is willing to pay 13.5 RUB extra for an SV located one-minute-walk closer. The value of θ implies that the elasticity of demand (total across all operators) at the equilibrium price is $-\frac{r}{\theta} = -1.60$; it also implies that, in the absence of competition with other operators, price markup could have been 4.28 RUB/min rather than currently estimated 0.69. The value of γ means that a 1% increase in travel time reduces the number of customers considering travel by 2%.

Area of Moscow region	Inside Garden ring	Between Garden and 3rd ring	Between 3rd ring and MKAD	outside MKAD
% of Delimobil home area	1.83	5.99	65.23	26.94
Observed % of vacant SVs	4.39	8.88	56.21	30.52
Predicted % of vacant SVs	3.08	7.82	61.86	27.24

Table 4: Observed vs. predicted distribution of vacant sample vehicles over space

7.3. Goodness of fit

The expected number of vacant sample vehicles in zone i is proportional to $\frac{f_i}{\sum_{j\neq i} q_{ij}}$, where f_i is the probability that a vehicle cycle originates at i, defined by (6), while $\frac{1}{\sum_{j\neq i} q_{ij}}$ is the expected vacancy duration. The correlation between this theoretical and empirically observed vacancy distributions across the 8778 Delimobil zones is as high as 80.6%. Note that the model of this paper ignores the daily cycle of vehicle use that tends to concentrate vacant vehicles in the center during working hours and in the periphery at night; the gooodness-of-fit would be even higher if this cycle was accounted for. Table 4 compares predicted and observed distributons of vacancy across four areas of Moscow, arranged from most central to most peripheral. The density of vacancies in the city center is underestimated, perhaps because the model ignores the hub-and-spoke road pattern, reducing the travel costs around the city center. At the same time, the density in the periphery, the most relevant for the counterfactual analysis of section 8, is predicted accurately. Figure 5 illustrates how the model fits the distrubution of trip distances, given the empirically observed distribution of origin locations. In such setting, the model predicts the mean geodesic distance to be 8.91 km, while under model-predicted distribution of origin locations, the mean distance is 10.8km. Empirically, mean geodesic trip distance is 8.84km.

The gross profit flow calculated according to (7) is $\phi = 0.560$, which is remarkably close to 0.583 calibrated in section 6.5.

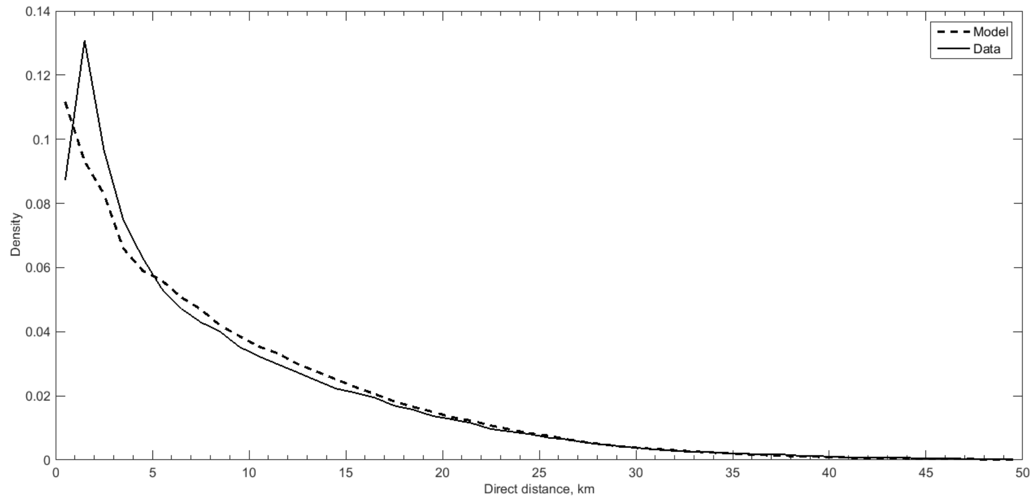

Figure 5: Distribution of trip geodesic distance, model vs. data

8. Counterfactual expansion of home area

8.1. Setup

This section studies a hypothetical new SV operator, *new operator* henceforth, that competes with existing operators, *old operators* henceforth, and makes use of optimal pricing derived in section 3.2.1. Introducing zones scores enables the new operator to expand its home area to all zones where any travel demand exists. Specifically, the new operator serves all 8778 *old* zones (as defined by Delimobil home area), as well as all 22041 *new* zones within the Moscow area with nightlight radiance between 10 and 100, as discussed in section 5.2. Figure 6 illustrates the counterfactual service area. The new operator is assumed to have a negligible market share, meaning that pricing policies of old operators remain unchanged.

Because the computational method of section 3.3 deals with the matrix q_{ij} of departure rates to/from every zone, calculations at the zone level would require operations with matrices containing almost one billion elements. To reduce computational burden, I clusterize the zones, with smaller clusters corresponding to higher-radiance areas. Table 5 details the cluster definition. At total of 3314 such *travel clusters* were identified, and all travel demand

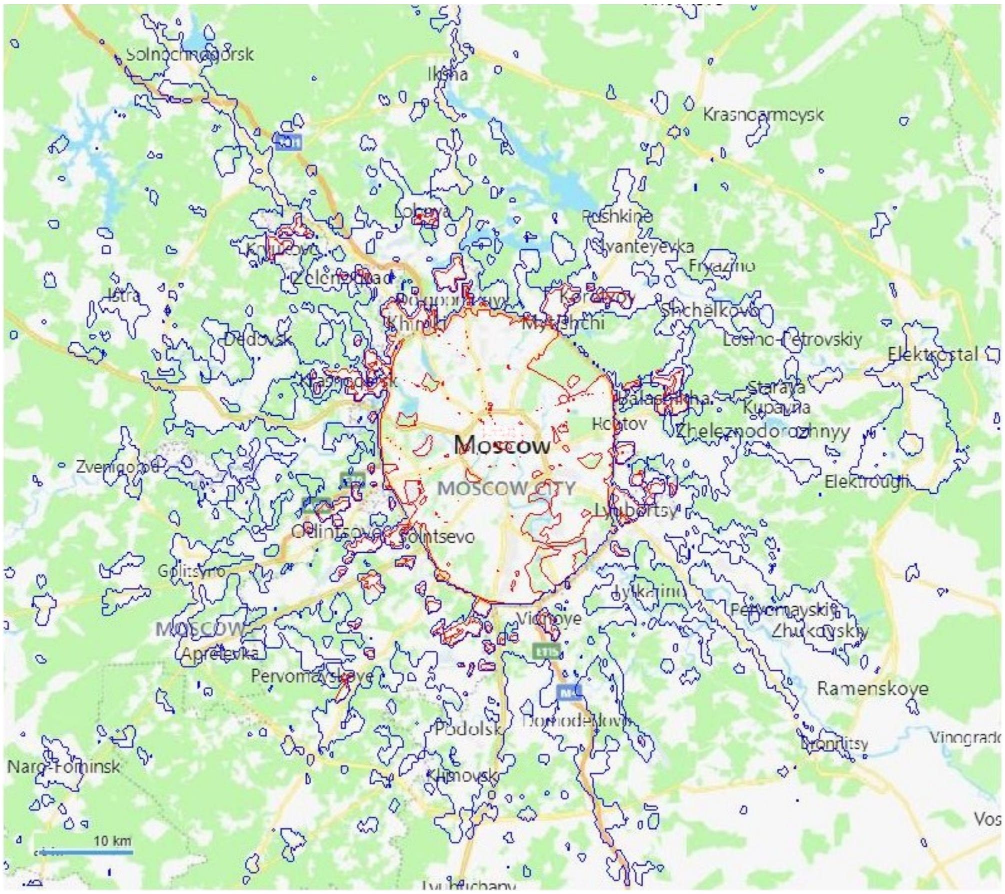

Figure 6: Counterfactual home area. Map source: Bing.

Nightlight radiance within cluster	≥ 100	$[50, 100)$	$[25, 50)$	< 25
Maximum cluster dimensions (# of zones)	2×4	4×4	4×8	8×8

Table 5: Travel cluster definitions

calculations were done at the level of these clusters.

Section 3.2.2 argues that dependence of a trip price on density of competing vehicles $\tilde{\mu}_{ij}$ should be reduced in order to prevent trip arbitrage by the customers. Within the old home area, we assume markup m_{ij} in (17) depends on some average $\hat{\mu}$ rather than actual $\tilde{\mu}_{ij}$: $m_{ij} = \frac{w\theta}{w+2\hat{\mu}\theta h_{ij}}$. To calculate $\hat{\mu}$, I average estimated density of vacant vehicles across the old zones, using the estimated distribution of cycle origins f_i as weights.

Because there is no competition in the new home area, trips with origin and/or destination in this area should optimally be priced with higher markups. For each zone i, define h_i^m the minimum travel time between zone i and any zone within old home area; naturally, $h_i^m = 0$ if i itself is in old home area. For every origin i and destinaton j, define $h_{ij}^D = \max\{h_{ij} - h_i^m - h_j^m, 0\}$ the maximum possible duration of travel within old (i.e. competitive) home area. The markup m_{ij} in (17) is then assumed to equal its competitive low level for h_{ij}^D minutes of the trip within the old home area, and to monopolist level (i.e. to θ) for the remainder of the trip:

$$m_{ij} = \frac{w\theta}{w + 2\hat{\mu}\theta h_{ij}^D}\frac{h_{ij}^D}{h_{ij}} + \theta\left(1 - \frac{h_{ij}^D}{h_{ij}}\right).$$

The old operators, which compete with the new operator in the old home area, do not practice any zone scores. Therefore, if the new operator introduced unequal zone scores within this area, trip arbitrage may emerge, with customers switching operators for different legs of their trip. To prevent that, we will assume that the entire old home area is the numeraire zone with zero score.

In the new home area, the zone scores R_i are made location-specific. Because R_i are optimization parameters, I introduce *pricing clusters* that are larger than travel clusters, to reduce dimensionality of the optimization problem. Specifically, pricing clusters also depend on nightlight radiance, and contain up to 2×2 travel clusters. E.g. pricing clusters with radiance below 25 can be as large as 16×16 zones.

8.2. Results

Following the computational method of section 3.3, I find that the new operator's gross profit can be as high as $\phi = 2.43$ RUB/min, which is 4.3 times as high as the estimated in section 7.3 gross profit of old operators. Most of this profit is earned from trips to/from new home area, where no competition exists. Because the new operator has higher opportunity cost ϕ of its vehicles, its price (17) for a typical trip inside the old home area tends to be higher than that of competitors. Specifically, while old operators ask an estimated 6.85 RUB per minute, the new operator's rate ranges from $c + \phi = 7.76$ RUB/min for long trips inside old home area to the monopolist rate of $c + \phi + \theta = 12.04$ RUB/min for short trips, plus the fee of ϕ for the reservation time.

Although the new home area has considerably less economic activity and therefore less travel demand, lack of competition makes many locations more attractive than the old home zone (i.e. than Moscow itself). Specifically, the optimal zone scores range from +852 RUB (13.5 USD in Dec.2019) in the downtown of Noginsk, a town east of Moscow, to -780 RUB in a village at the boundary of the studied area. In total, 6634 (30%) of all new-home-area zones have positive score, while the score of others is negative. As expected, the zone score strongly depends on travel demand as proxied by nightlight radiance, with 60.5% correlation. Figure 7 illustrates the zone scores for North-Eastern suburbs of Moscow.

Not surprisingly, high attractiveness of some new zones will have a strong effect on vacant vehicle distribution. In counterfactual equilibrium, a vehicle of the new operator spends only 26.3% of its vacancy time within the old home area. Table 6 details the counterfactual distribution of vacancy locations. Operations are shifted mainly to new zones with radiance above 50, which are typically located in centers of towns around Moscow.

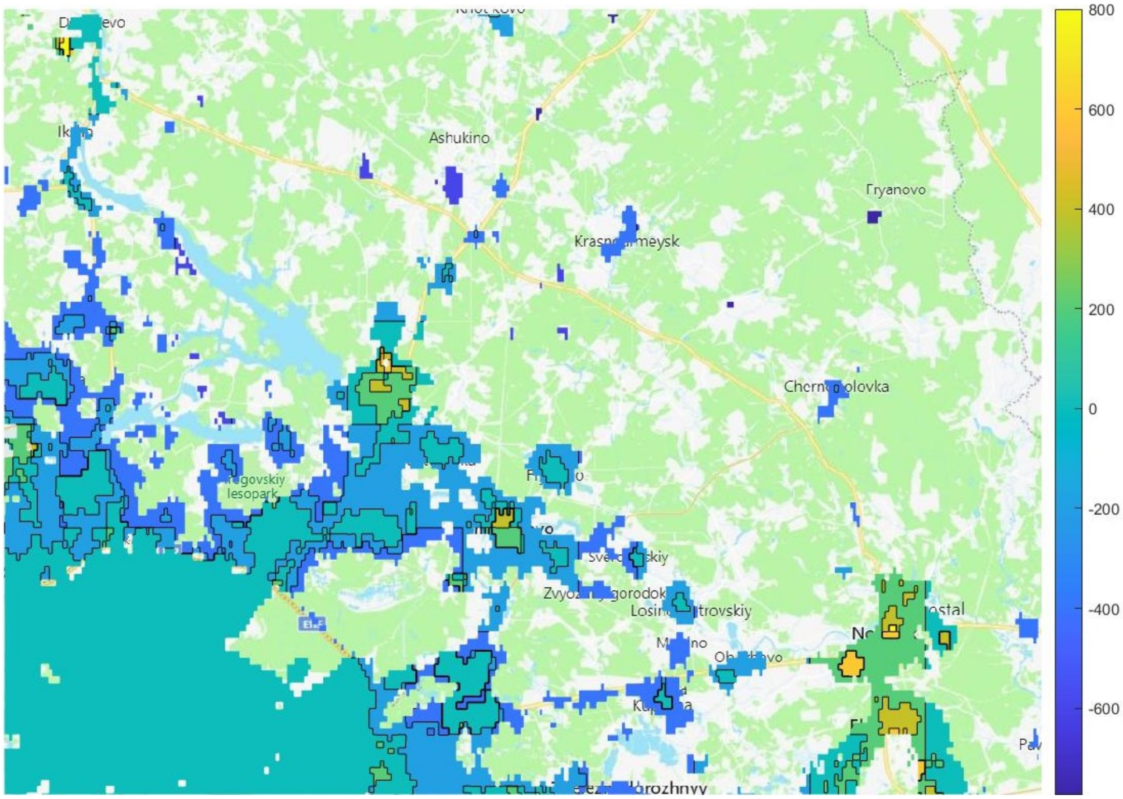

Figure 7: Zone scores, North-East of Moscow. Map source: Bing.

Area	Old home	New home area by radiance		
	area	≥ 50	$[25, 50)$	$[10, 25)$
% of territory	28.26	4.49	17.14	50.11
Predicted % of vacant SVs	26.34	47.75	17.26	8.65

Table 6: Counterfactual distribution of vacant vehicles over space

9. Conclusion

This paper introduces the first, to the best of the author's knowledge, theory of optimal shared-vehicle pricing based on a novel model of SV market. The paper shows that introduction of location-specific pricing for shared vehicles may help to dramatically expand the geography of service, eventually making it nationwide. Availability of shared vehicles scattered around the country may reduce individual vehicle ownership, making it primarily a rural phenomenon. Reduced vehicle ownership, in turn, would have a positive effect on household welfare, as purchasing a car is typically the second-largest expense of a household. Fewer vehicles would also reduce parking demand, making cities less congested.

In low-density areas, however, vacant shared vehicles would be too sparse to become a viable transportation option for local everyday trips. But they still could be used to longer-distance trips, especially for trips to large cities with high cost of parking an individual vehicle.

The model can be extended in a number of ways. A daily cycle of travel demand could be accounted for. This would open the possibility of time-varying zone scores to encourage trips to locations where the number of reservations is expected to increase in the near future.

Trip distances and durations could be calculated more accurately, accounting for the actual road network. This could also lead to a better method of zone score calculation.

After suburban service is introduced, nightlight data could be abandoned, in favor of collected trip data, to better predict future trip demand.

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