The economics of parking occupancy sensors

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Abstract

Parking occupancy sensors are devices that assist search of vacant parking. The interplay between two government policies, installation of sensors and pricing parking, is studied. When parking is congested and pricing is optimal, installation of sensors raises the price, increases turnover of parking. If price discrimination is considered, sensored parking should be cheaper than non-sensored. To achieve optimal search of vacant parking, it is sufficient to equip only a fraction of parking with sensors. Underpriced parking may dampen the sensor installation incentive, relative to the incentive to build extra parking. Nevertheless, in absolute terms the sensor installation incentive is substantial even with free parking.

Keywords: parking occupancy sensor, pricing parking

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1. Introduction

Parking occupancy sensors (sensors henceforth) are small devices built into the surface of automobile parking bays. Their function is to detect the presence of a vehicle in that bay and to live-feed the information, wirelessly, to the motorists searching for parking. Such information makes the search for parking more directed and thereby decreases the search time, pollution, accidents, and traffic.

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The sensor technology, however, is not free. For example, San Francisco's SFPark, a project that deployed over 5000 sensors throughout the city center, required about $27 million of funding over two years. Barcelona's Urbiotica cost was between $200 and $400 per sensor to install (Ross (2011)). It is therefore important to understand the economics of parking sensor installation, which is the primary focus of this paper.

Specifically, the paper analyzes the interplay between two government policies: installation of parking sensors and pricing parking. While the latter problem has been extensively discussed in many studies, it has always focused either on homogenous supply of parking (as in Glazer and Niskanen (1992), Arnott and Inci (2010), Zakharenko (2016)) or on exogenous heterogeneity of parking supply (Anderson and de Palma (2004), Kobus et al. (2013), Inci and Lindsey (2015)). Because installation of sensors is a government choice, sensor technology creates an endogenous heterogeneity of parking supply. The goal of this paper is to understand how such heterogeneity affects optimal pricing, especially in the presence of heterogenous demand for parking, and how sensor installation itself is optimally affected by pricing. This paper shows that the mutual dependence of the two government policies can be substantial.

To the best of my knowledge, this is the first study of economic aspects of parking occupancy sensors.

Methodologically, this paper develops a theoretical dynamic model of demand and supply for parking that allows heterogeneity of motorists with respect to their desired duration of parking and their value of a parking session. The model incorporates a costly search for a vacant parking space that may or may not be assisted by occupancy sensors. Throughout most of the paper, motorists decide whether to search for parking endogenously; their decision is affected by the difficulty of search and by the monetary cost of parking. The resulting equilibrium can be managed by the government by means of price regulation, installation of additional parking sensors, and providing more parking space.

Several major questions are answered. Section 3 studies how installation of sensors affects the optimal (unform) price for parking. Section 4 analyzes what can be gained by price discrimination. Section 5 shows that only a fraction of all parking bays should be equipped with sensors. Section 6 studies how the price for parking affects the economic incentives to install sensors – in absolute terms and relative to the incentives to expand parking space.

2. The model

The model of parking is elaborated from Zakharenko (2016). This is a model of a city area with geographically homogenous demand and supply of parking, set up in continuous time. For transparency of the arguments, all aggregate parameters of the current model are time-invariant.

2.1. Supply of parking

There is a continuum of parking space of measure $2N$. The standard method to model search for parking is to assume that the inflowing travelers randomly and independently sample parking bays, until a vacancy is found. Such assumption is found, among others, in Zakharenko (2016), Geroliminis (2015), Anderson and de Palma (2004), and Arnott and Rowse (1999). Recently, Arnott and Williams (2017) criticize this approach, which they call a *binomial approximation*, for its neglect of spatial correlation of parking occupancy. For the purposes of this paper, however, the analytical tractability advantage of the binomial approximation outweighs its drawbacks.

At the same time, to emphasize that information fed by the sensors may affect the motorist search path, we modify the standard binomial-approximation search as follows. We will assume that all parking bays are bundled into *sets* of two units; each searching motorist randomly and independently draws a set, rather than a single bay, and can choose which one (and only one) of the two bays within a set to inspect for vacancy. If a bay is equipped with a sensor, its occupancy status is known without inspecting it.

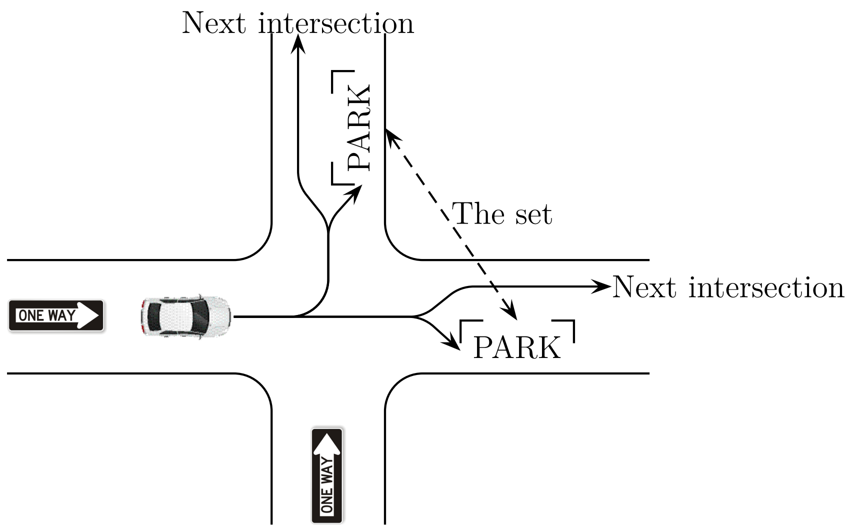

Figure 1: A spatial illustration of search for parking. Vehicle image courtesy of Macrovector / Freepik.

This model of search for parking has the following spatial interpretation. Imagine a city with a grid of North-South and East-West streets. All streets are one-way. Then, at each intersection, a motorist faces a set of two choices: to continue straight or to turn to the crossing street. For simplicity, on all streets there is only one parking bay per block. The motorists cannot plan more than one block ahead and treat all intersections as spatially independent from each other. This spatial model is illustrated on Figure 1.

Different parking bays are assumed to be sufficiently proximate to each other so the travelers get the same utility from being parked at either of them. Because each set has two parking bays, the total mass of sets is N.

2.2. Demand for parking

At each instant of time, there is a flow of newly *emerging* motorists who decide instantly whether they are willing to (i) travel to the city area in question by car and search for parking, or (ii) use an outside opportunity, such as another type of transportation or no travel at all. For those who choose to drive and park, the time elapsed from the decision to use car to the beginning of the search for parking is immaterial, and is assumed to be zero. Thus, those who decide to use a car are said to instantly *enter* the parking zone. They search for parking until a vacancy is found. The time is normalized to that it takes one unit of time to inspect one parking bay, and the opportunity cost of a unit of search time is c.[2]

The emerging motorists differ in their desired duration of parking τ and their value of a unit of parking time v. By $A(\tau, v)$ we denote the flow of emerging motorists with characteristics τ and v.

After a decision to enter was made, motorists choose the optimal search strategy which minimizes the sum of their opportunity costs of search time and monetary costs of a parking session.

If a traveler chooses to stay out, her outside opportunity is normalized to zero.

3. The optimal unform price for parking

This section studies how installation of one sensor per set of two parking bays affects the optimal price regulation of parking. In this section, we restrict ourselves to the case when all parking costs the same. We consider separately the scenarios with no sensors at all and one sensor per set of parking bays.

3.1. Equilibrium entry decisions

Denote by q the *congestion level*, i.e. the probability that a particular draw of a parking set will not result in a parking session. Under the unform pricing, we show later that all motorists employ the same search strategy and thus the congestion level is the same for everyone. Then, the expected duration of search is $\frac{1}{1-q}$ so the opportunity cost of search is $\frac{c}{1-q}$. Assuming the price for parking is p, a motorist with desired parking duration τ and

[2]It is never optimal to abandon search once it has started, due to three assumptions: (i) the model aggregates are time-invariant; (ii) motorist utility does not depend on the time when the parking session has begun, only on the total duration of parking; (iii) the opportunity cost of search, per unit of time, is constant. Given these assumptions, the expected utility from continuing search does not depend on previous search history or on the time of the day.

value of parking v will incur a monetary cost of $p\tau$ while enjoying the utility of $v\tau$. Thus, an emerging motorist will choose to search if her value per unit of parking time exceeds

$$v^*(p, q, \tau) = p + \frac{1}{\tau}\frac{c}{1-q}. \tag{1}$$

The total flow of motorists entering the system is then $\int_\tau \int_{v^*(p,q,\tau)}^\infty A(\tau, v)\mathrm{d}v\mathrm{d}\tau$.

Denote by Q the *occupancy level*, i.e. the proportion of all parking bays that are occupied in equilibrium. It is defined by

$$Q \equiv \frac{\int_\tau \int_{v^*(p,q,\tau)}^\infty A(\tau, v)\tau\mathrm{d}v\mathrm{d}\tau}{2N}, \tag{2}$$

where the numerator is the total mass of vehicles parked at a given instant of time, while the denominator is the capacity.

Next, we analyze and compare the equilibria with and without parking sensors.

3.2. No sensors

Without information supplied by sensors and with uniform price for parking, a searching motorist who has drawn a set of two parking bays chooses to inspect the one with the lowest expected occupancy level. In equilibrium, clearly, both bays within a set should have the same occupancy level, and each motorist should randomize with equal probabilities. But then the model of search is identical to the standard binomial-approximation search, and is a special case of Zakharenko (2016). Particularly, the congestion level q is equal to the occupancy level Q, turning the definition of occupancy (2) into the equation for the demand for parking:

$$Q = \frac{\int_\tau \int_{v^*(p,Q,\tau)}^\infty A(\tau, v)\tau\mathrm{d}v\mathrm{d}\tau}{2N}. \tag{3}$$

The optimal price, as elaborated in Zakharenko (2016), is equal to the expected externality that a parked vehicle has on searching vehicles, and is given by

$$p = \frac{c \int_{\tau} \int_{v^*(p,Q,\tau)}^{\infty} A(\tau, v) \mathrm{d}v \mathrm{d}\tau}{2N(1-Q)^2}. \tag{4}$$

The optimal price is a product of three multipliers:

(i) $\frac{\int_{\tau} \int_{v^*(p,Q,\tau)}^{\infty} A(\tau,v)\mathrm{d}v\mathrm{d}\tau}{1-Q}$ is the total mass of searching motorists. It is itself a product of the flow of newly entering motorists, $\int_{\tau} \int_{v^*(p,Q,\tau)}^{\infty} A(\tau, v)\mathrm{d}v\mathrm{d}\tau$, and the expected search duration for each of them, $\frac{1}{1-Q}$.

(ii) $\frac{c}{1-Q}$ is the expected cost of continuing search, if a bay in question was occupied,

(iii) $\frac{1}{2N}$ is the marginal impact of one occupied bay on the congestion level.

In other words, (ii) is the externality of a parked vehicle on a searching vehicle that visited the same bay, due to the need to continue search, while (iii) is a measure of the probability that a particular searching motorist visits a particular parking bay. The product of (i) and (iii) is also the expected number of searching motorists that arrive at (i.e. inspect for vacancy) the bay in question, per unit of time.

For the transparency of the argument that follows, we impose the following regularity condition.

Assumption 1. *The right-hand side of (4) is increasing in Q.*

The assumption essentially states that a decrease in the flow of new entries (the numerator of (4)) following the increase in occupancy Q is small enough and cannot lower the optimal price.

The relationship between p and Q is negative in (3). This relationship is positive in (4) due to Assumption 1. Assuming the value of p in (4) is small enough at $Q = 0$, and because it increases to infinity, there is a unique solution of the system (3,4) which is illustrated on Figure 1 of Zakharenko (2016).

3.3. One sensor per set

Suppose now that one of two bays in each set is equipped with a parking sensor. With uniform price, the searching motorists minimize their search cost and thus adopt what we call the *fast strategy* of search: take the sensored bay if it is vacant; inspect the non-sensored bay otherwise. Thus, a motorist will *not* be able to park after drawing a set only if both bays in the set are occupied. Therefore, sensors, by making the search more directed, allow the congestion q to be lower than occupancy Q.

3.3.1. Calculating congestion

Given the congestion level q, the expected duration of search is equal to $\frac{1}{1-q}$, hence the arrival rate of searching motorists per set of two parking bays is

$$E \equiv \frac{\int_\tau \int_{v^*(p,q,\tau)}^\infty A(\tau, v)\mathrm{d}v\mathrm{d}\tau}{N(1-q)}. \tag{5}$$

Also denote by X the intensity of departures from occupied parking bays; it is equal to the inverse of the expected duration of parking,

$$X \equiv \frac{\int_\tau \int_{v^*(p,q,\tau)}^\infty A(\tau, v)\mathrm{d}v\mathrm{d}\tau}{\int_\tau \int_{v^*(p,q,\tau)}^\infty A(\tau, v)\tau\mathrm{d}v\mathrm{d}\tau}. \tag{6}$$

To calculate the equilibrium congestion level, denote by $q_{SN}, S \in \{0,1\}, N \in \{0,1\}$ the probability that a set is in occupancy status SN, where S (N) is the occupancy indicator of the sensored (non-sensored) bay. Thus, a motorist cannot park only in the 11 state, hence q_{11} is equal to congestion q. The steady-state transition between the four occupancy states is found from the following:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \dot{q}_{00} \\ \dot{q}_{01} \\ \dot{q}_{10} \\ \dot{q}_{11} \end{pmatrix} = \begin{pmatrix} -E & X & X & 0 \\ 0 & -X-E & 0 & X \\ E & 0 & -X-E & X \\ 0 & E & E & -2X \end{pmatrix} \begin{pmatrix} q_{00} \\ q_{01} \\ q_{10} \\ q_{11} \end{pmatrix} . \tag{7}$$

For example, the second column of the 4-by-4 transition matrix in (7) indicates how the transition from the 01 state (vacant sensored bay, occupied non-sensored bay) occurs. Searching motorists arrive with intensity E and occupy the sensored bay, moving the system into the 11 state. Those parked at the non-sensored bay depart with intensity X, moving the system into the 00 state.

The system (7) has the following solution:

$$\begin{pmatrix} q_{00} \\ q_{01} \\ q_{10} \\ q_{11} \end{pmatrix} = \frac{1}{(E+X)(E^2+2EX+2X^2)} \begin{pmatrix} 2X^2(E+X) \\ E^2X \\ EX(E+2X) \\ E^2(E+X) \end{pmatrix} , \tag{8}$$

and therefore the steady-state level of congestion is given by:

$$q \equiv q_{11} = \frac{E^2}{2X^2+2EX+E^2} \tag{9}$$

Note that the arrival intensity E, as defined by (5), is itself a function of q. By solving the system (5,9), dropping the irrelevant solution, and by substituting (6), we obtain the following equilibrium congestion level:

$$q(Q) = Q + \frac{1}{2} - \frac{1}{2}\left(4Q(1-Q)+1\right)^{\frac{1}{2}} , \tag{10}$$

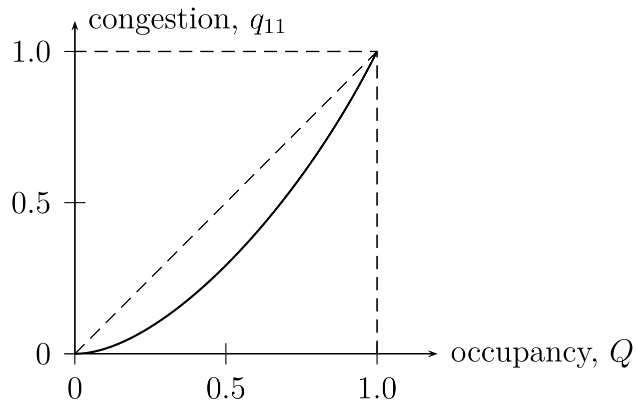

Figure 2: Congestion as function of occupancy with sensored parking

where the equilibrium occupancy level Q as defined by (2). Figure 2 illustrates the relationship. Congestion is lower than occupancy except at the corners. Also note that, when occupancy is zero, a marginal increase in occupancy has a lower-order-of-magnitude effect on congestion. This is because, when only a small fraction of parking is occupied, sensors allow the searchers to navigate around the occupied spots at no additional time cost.

At the same time, a marginal change in occupancy near $Q = 1$ leads to a double change in congestion. This is because when almost all parking is occupied, the chance that a set has both bays vacant is negligibly small, so the number of vacant bays is approximately equal to the number of sets with one vacant bay. But then, a small increase in occupancy rate would mean a double increase in the proportion of fully occupied sets (because the number of sets is half the number of parking bays), so the congestion rate should also increase by a double amount.

3.3.2. Demand for parking

By substituting $q(Q)$ given by (10) into the definition of occupancy (2), we obtain the following function for the demand for parking:

$$Q = \frac{\int_\tau \int_{v^*(p,q(Q),\tau)}^{\infty} A(\tau, v)\tau \mathrm{d}v \mathrm{d}\tau}{2N}. \tag{11}$$

Because congestion is lower than occupancy, $q(Q) \leq Q$, with strict inequality for interior Q's, the right-hand side of (11) is larger than that of (3) for any values of $p \geq 0$ and $Q \in (0, 1)$. For equality in (3) to be true, we must conclude that there is more occupancy demanded Q under the same price p when the sensors are installed. Intuitively, sensors facilitate the search and thus induce more entry.

3.3.3. Optimal price

To calculate the socially optimal price, we reiterate the three multipliers of the price listed below (4).

(i) The total mass of searchers is $\frac{\int_\tau \int_{v^*(p,q(Q),\tau)}^{\infty} A(\tau,v)\mathrm{d}v\mathrm{d}\tau}{1-q(Q)}$,

(ii) the expected time opportunity cost of a searching motorist is $\frac{c}{1-q(Q)}$,

(iii) the marginal impact of a parked vehicle on congestion q is the product of (a) its marginal impact on occupancy Q, still equal to $\frac{1}{2N}$, and (b) the marginal impact of occupancy on congestion, $q'(Q) = 1 + (2Q - 1)(4Q(1 - Q) + 1)^{-\frac{1}{2}}$.

Thus, the optimal price can be defined as

$$p = \frac{c \int_\tau \int_{v^*(p,q(Q),\tau)}^{\infty} A(\tau, v)\mathrm{d}v\mathrm{d}\tau}{2N(1 - q(Q))^2} \left[1 + (2Q - 1)(4Q(1 - Q) + 1)^{-\frac{1}{2}}\right]. \tag{12}$$

Note that, given the same values of p and Q, the ratio in (12) is smaller than the same ratio in (4), due to $q(Q) \leq Q$ and Assumption 1. But most of the discrepancy between (12) and (4) is due to the term in square brackets in (12): it rises from zero to two as Q increases from zero to one. Thus, we may conclude that the socially optimal price is zero when $Q = 0$, and is asymptotically twice as high as that in (4) when occupancy increases to unity.

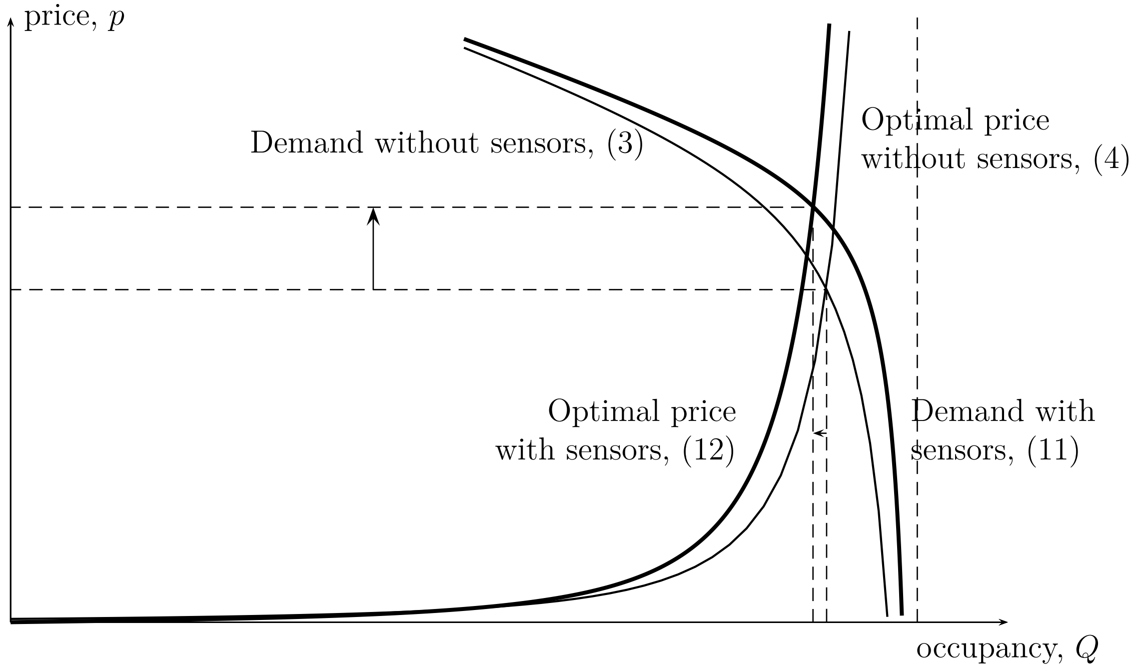

Figure 3: Effect of sensor installation on demand and optimal price

3.3.4. Effect of sensorization on equilibrium

The above discussion implies that the effect of sensorization on equilibrium occupancy Q and optimal price p can be ambiguous and depends on the initial occupancy level. The most interesting case is when occupancy is very high and the sensors make the most economic sense. With non-sensored occupancy Q being close to one, the congestion $q(Q)$ is also close to one, hence the demand functions (3) and (11) are close to each other. At the same time, the price p is nearly twice as high in the sensored case, as evidenced by Section 3.3.3. Therefore, the new equilibrium with sensors is characterized by a higher price for parking, a lower occupancy Q, and a still lower congestion level $q(Q)$. The change in equilibrium parameters is illustrated on Figure 3.

These changes also induce a shift in the structure of demand for parking. A higher price p will result in fewer arrivals of motorists with large parking durations τ, for whom the monetary part of parking costs dominates. At the same time, lower congestion q will result in more arrivals of those parking short-term, because for them the relief in the opportunity time cost of search outweighs the extra monetary cost of parking. Thus, installation of sensors in congested areas, combined with optimal uniform pricing, will increase the turnover of parking.

4. Price discrimination

4.1. No sensors

Even without the sensor technology, the benefits of price discrimination of parking have not been properly studied. The predecessor of the current model, Zakharenko (2016), has only considered a uniform price so all parking sites are ex-ante equally attractive to those who search for parking. What if the two parking bays within a set are priced differently? Some studies, e.g. Anderson and de Palma (2004), Arnott and Rowse (2009) have considered geographic segregation of parking, but no study to my knowledge has discussed whether it is beneficial to segregate parking *at the same location*. We now analyze this question, as a prerequisite to understanding price discrimination between sensored and non-sensored parking.

Divide all parking bays within each set into two types, denoted 0 and 1. For example, type-0 parking could be that on North-South streets while type-1 on East-West streets. Suppose first that the socially optimal price (4) is the same for both types, $p_1 = p_0$. Then, the congestion levels for the two parking types must be the same too, $q_0 = q_1$, otherwise searching for the lower-q type parking would be easier for all motorists and no one would demand the other. Then, all parkers are completely indifferent between the two zones, and can be directed to any of them without any effect on welfare, as long as the equality of congestion is preserved.

Suppose all parkers whose duration of parking τ is under some τ^* search for type-0 parking, while the rest of parkers search for type-1. The cutoff τ^* is determined by equality of occupancy/congestion in the two zones. In math, denote by $I_i(\tau)$ the probability that a motorist with parking duration τ searches in zone $i \in \{0, 1\}$. We have that $I_0(\tau) = 1$ iff $\tau \leq \tau^*$ and $I_1(\tau) = 1$ iff $\tau > \tau^*$. Denote by τ_i the mean duration of type-i parking,

$$\tau_i \equiv \frac{\int_\tau \int_{v=v^*(p_i,q_i,\tau)}^{\infty} A(\tau, v)I(\tau)\tau \mathrm{d}v\mathrm{d}\tau}{\int_\tau \int_{v=v^*(p_i,q_i,\tau)}^{\infty} A(\tau, v)I(\tau)\mathrm{d}v\mathrm{d}\tau}. \tag{13}$$

Obviously, $\tau_0 \leq \tau^* \leq \tau_1$, with at least one inequality being strict when there is non-zero variance of τ. The occupancy in zone i is determined by (cf.(3))

$$q_i = \frac{\int_\tau \int_{v=v^*(p_i,q_i,\tau)}^{\infty} A(\tau, v)I(\tau)\tau \mathrm{d}v\mathrm{d}\tau}{N} \underbrace{=}_{\text{cf.(13)}} \frac{\tau_i \int_\tau \int_{v=v^*(p_i,q_i,\tau)}^{\infty} A(\tau, v)I(\tau)\mathrm{d}v\mathrm{d}\tau}{N}. \tag{14}$$

The optimal price in zone i can be derived from (4) using (14) as follows:

$$p_i = \frac{cq_i}{\tau_i(1 - q_i)^2}. \tag{15}$$

Because we have that $q_1 = q_0$, the equality of optimal prices $p_1 = p_0$ can be held only when parking duration is homogenous so that $\tau_1 = \tau_0$. But in case of heterogeneity of τ, we have that $\tau_0 < \tau_1$, hence a welfare improvement is possible, hence the initial assumption $p_1 = p_0$ is not socially optimal.

This finding suggests that, even without parking sensors, it is beneficial to make a fraction of parking capacity more expensive. Then, short-term parkers benefit from shorter search of vacancy in the expensive zone, while the long-term parkers benefit from the lower cost of the cheap zone. To the best of my knowledge, this result is novel by itself: no prior study has discussed the benefits of price discrimination for parking that is geographically homogenous.

We use this result as a starting point for the analysis that follows.

4.2. Partly sensored parking

4.2.1. Assumptions

This section analyzes whether price discrimination can help achieve any welfare improvements when one parking bay in each set is equipped with a sensor. To improve mathematical tractability, we will consider only marginal price differences between sensored and non-sensored parking. We also make the following simplifying assumption.

Assumption 2. *Entry is exogenous: price variations do not affect motorist entry decisions, so the entry lower bound v^* depends only on τ.*

This assumption allows us, among other things, to simplify notation by denoting $A(\tau) \equiv \int_{v^*(\tau)}^{\infty} A(\tau, v)\mathrm{d}v$ the mass of all newly entering motorists desiring to park for τ units of time.

4.2.2. Search strategies and intuition of optimal discrimination

With exogenous entry, the only choice of the motorists is to optimize their strategy of search for parking. Two strategies can be distinguished. The fast strategy of search was introduced in Section 3.3 and minimizes the time spent searching. This strategy is suitable for those who park for shorter periods of time, i.e. for whom the monetary cost constitutes a relatively small fraction of the total cost of parking, and the incentive to minimize the search time dominates.

The *economy* strategy is to always search for the cheapest vacant bay, i.e. do not attempt to park at the more expensive bay even if it is known to be vacant. The economy strategy is suitable for those parking long-term, i.e for those with the monetary savings of cheaper parking exceeding the opportunity cost of extra search time.

Which of the two types of parking, sensored or non-sensored, should be cheaper? Before proving the result formally, we discuss the economic intuition. Suppose the price difference is small so the vast majority of motorists use the fast strategy of search. The philosophy of optimal pricing is to make the price for parking equal to the externality that a parked vehicle has on subsequently arriving vehicles. But with majority of subsequent arrivals using the fast strategy, they will not park only if both bays in a set are occupied. Therefore, the externality of a vehicle parked at bay of type $i \in \{S, N\}$ is proportional to the conditional probability that the other bay is occupied, too. In math, the externality of being parked in the sensored bay is proportional to $\Pr(N = 1|S = 1) = \frac{q_{11}}{q_{11}+q_{10}}$ while the externality of being parked in the non-sensored bay is proportional to $\Pr(S = 1|N = 1) = \frac{q_{11}}{q_{11}+q_{01}}$. If the vast majority of motorists use the fast strategy, the values of q_{SN} are similar to those found in (8), where all motorists use such strategy. But then, because the sensored bays are taken before the nonsensored ones, we have that $q_{10} > q_{01}$, which means that the externality of parking at a sensored bay is lower than the same externality at a non-sensored bay, $\Pr(N = 1|S = 1) < \Pr(S = 1|N = 1)$, and therefore sensored bays should be cheaper. The analysis that follows confirms this intuition formally.

4.2.3. Formal analysis

Suppose the cost of parking is p at a sensored bay and $p + \epsilon$ at a non-sensored bay. To achieve analytical tractability of the results, we will focus on a marginal discrimination, i.e. on values of ϵ converging to zero.

For a fast-strategy searcher, the expected total cost of parking for τ units of time is then $\frac{c}{1-q_{11}(\epsilon)} + p\tau + \epsilon\tau\frac{q_{10}(\epsilon)}{1-q_{11}(\epsilon)}$, where $q_{SN}(\epsilon)$ is the equilibrium probability of a set to be in state SN given price discrimination ϵ, $\frac{1}{1-q_{11}(\epsilon)}$ is the expected search duration under the fast strategy, and $\frac{q_{10}(\epsilon)}{1-q_{11}(\epsilon)}$ is the fraction of fast-strategy motorists who eventually park at a non-sensored bay.

An economy-strategy motorist searches only for sensored bays because they are cheaper. The expected total cost of parking is $\frac{c}{1-q_{11}(\epsilon)-q_{10}(\epsilon)} + p\tau$, hence, relative to the fast search strategy, the opportunity time cost of search is higher but the expected monetary expense is lower. By comparing the parking costs under the fast and the economy strategies, we conclude that the latter will be chosen by those with $\tau > \tau^*(\epsilon)$ such that

$$\tau^*(\epsilon) \equiv \frac{c}{\epsilon}\frac{1}{1 - q_{10}(\epsilon) - q_{11}(\epsilon)}. \tag{16}$$

As ϵ converges to zero, an increasing number of motorists follows the fast search strategy so the values of $q_{SN}(\epsilon)$ converge to those defined by (8), so the cutoff parking duration $\tau^*(\epsilon)$ is asymptotically inverse of ϵ and therefore rises to infinity.

To calculate how price discrimination affects the probability distribution of occupancy states, we reiterate Section 3.3.1 to account for the existence of two search strategies. Observe that the mass of newly entering economy-strategy motorists equals $\delta(\epsilon) \equiv \int_{\tau^*(\epsilon)}^{\infty} A(\tau)\mathrm{d}\tau$ while the mass of economy-strategy motorists parked at any given time is $\gamma(\epsilon) \equiv \int_{\tau^*(\epsilon)}^{\infty} A(\tau)\tau\mathrm{d}\tau$. As ϵ converges to zero, both of these quantities converge to zero, too; however, $\delta(\epsilon)$ converges at a faster rate than $\gamma(\epsilon)$. For proof, observe that the ratio of the latter to the former quantity is merely the mean parking duration of economy-strategy motorists, $\frac{\gamma(\epsilon)}{\delta(\epsilon)} = \frac{\int_{\tau^*(\epsilon)}^{\infty} A(\tau)\tau\mathrm{d}\tau}{\int_{\tau^*(\epsilon)}^{\infty} A(\tau)\mathrm{d}\tau}$, which converges to infinity as it cannot be less than $\tau^*(\epsilon)$. But then, in the asymptotic analysis of the equilibrium, we have that $\delta(\epsilon) = o(\gamma(\epsilon))$, which simplifies the analysis further.

Price discrimination changes the intensities of arrival to and exit from the parking bays. The intensity of arrival of economy-strategy motorists per parking set is the entry rate of such motorists $\delta(\epsilon)$, times their expected duration of search $\frac{1}{1-q_{10}(\epsilon)-q_{11}(\epsilon)}$, divided by the mass of parking sets: $E_E(\epsilon) = \frac{\delta(\epsilon)}{N(1-q_{10}(\epsilon)-q_{11}(\epsilon))} = o(\gamma(\epsilon))$. The intensity of arrival of fast-strategy motorists is $E_F(\epsilon) = E(\epsilon) - \frac{\delta(\epsilon)}{N(1-q_{11}(\epsilon))} = E(\epsilon) - o(\gamma(\epsilon))$, where (cf.(5))

$$E(\epsilon) \equiv \frac{\int_0^{\infty} A(\tau)\mathrm{d}\tau}{N(1 - q_{11}(\epsilon))}. \tag{17}$$

By $E = E(0)$ we will refer to the arrival intensity when there is no price discrimination.

The rate of exit from non-sensored bays, used only by fast strategy motorists, is the inverse of mean parking duration of such motorists:

$$X_N(\epsilon) = \frac{\int_0^\infty A(\tau)\mathrm{d}\tau - \delta(\epsilon)}{\int_0^\infty A(\tau)\tau\mathrm{d}\tau - \gamma(\epsilon)} = Y_N(\gamma(\epsilon)) + o(\gamma(\epsilon)), \tag{18}$$

where

$$Y_N(\gamma) = X + \frac{\int_0^\infty A(\tau)\mathrm{d}\tau}{\left(\int_0^\infty A(\tau)\tau\mathrm{d}\tau\right)^2}\gamma = X + \frac{X^2}{(1-q_{11})E}\gamma = X + \frac{X(E^2+2EX+2X^2)}{2E(E+X)}\gamma \tag{19}$$

and where the non-discrimination exit rate X is given by (cf.(6)) $X = \frac{\int_0^\infty A(\tau)\mathrm{d}\tau}{\int_0^\infty A(\tau)\tau\mathrm{d}\tau}$.

The rate of exit X_S from sensored bays is found similarly, accounting for the fact that such bays are used by all economy-strategy motorists, as well as by fraction $\frac{1-q_{10}(\epsilon)-q_{11}(\epsilon)}{1-q_{11}(\epsilon)}$ of fast-strategy motorists:

$$X_S(\epsilon) = \frac{\delta(\epsilon) + \frac{1-q_{10}(\epsilon)-q_{11}(\epsilon)}{1-q_{11}(\epsilon)}\left(\int_0^\infty A(\tau)\mathrm{d}\tau - \delta(\epsilon)\right)}{\gamma(\epsilon) + \frac{1-q_{10}(\epsilon)-q_{11}(\epsilon)}{1-q_{11}(\epsilon)}\left(\int_0^\infty A(\tau)\tau\mathrm{d}\tau - \gamma(\epsilon)\right)} = Y_S(\gamma(\epsilon)) + o(\gamma(\epsilon)), \tag{20}$$

where

$$Y_S(\gamma) = X - \frac{\int_0^\infty A(\tau)\mathrm{d}\tau}{\left(\int_0^\infty A(\tau)\tau\mathrm{d}\tau\right)^2}\frac{q_{10}}{1-q_{10}-q_{11}}\gamma = X - \frac{X(E+2X)}{2(E+X)}\gamma. \tag{21}$$

For a given ϵ, the matrix of transition between the occupancy states for a given set of parking bays is modified from (7) as follows (dropping the arguments of E_E, E_F, X_N, X_S):

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -E_E - E_F & X_N & X_S & 0 \\ 0 & -X_N - E_E - E_F & 0 & X_S \\ E_E + E_F & 0 & -X_S - E_F & X_N \\ 0 & E_E + E_F & E_F & -X_S - X_N \end{pmatrix} \begin{pmatrix} q_{00}(\epsilon) \\ q_{01}(\epsilon) \\ q_{10}(\epsilon) \\ q_{11}(\epsilon) \end{pmatrix}, \tag{22}$$

which results, among others, in the following probability that both bays in a set are occupied (dropping the argument of Y_N and Y_S):

$$q_{11}(\epsilon) = \frac{E(\epsilon)^2(E(\epsilon)+Y_N)}{Y_S Y_N(2E(\epsilon)+Y_S+Y_N)+E(\epsilon)^2 Y_S+E(\epsilon)Y_N(E(\epsilon)+Y_S+Y_N)+E(\epsilon)^2(E(\epsilon)+Y_N)}+o(\gamma). \tag{23}$$

At the same time, the arrival per set of bays, $E(\epsilon)$, is related to $q_{11}(\epsilon)$ as described by (17). From there, we can express $q_{11}(\epsilon)$ as a function of $E(\epsilon)$ and substitute it into (23). Then, redenoting $J(\gamma(\epsilon)) \equiv E(\epsilon)$, we have

$$\begin{aligned} G(J,\gamma) &\equiv 1 - \frac{1}{J}\frac{\int_\tau A(\tau)\mathrm{d}\tau}{N} \\ -&\frac{J^2(J+Y_N(\gamma))}{Y_S(\gamma)Y_N(\gamma)(2J+Y_S(\gamma)+Y_N(\gamma))+J^2Y_S(\gamma)+JY_N(\gamma)(J+Y_S(\gamma)+Y_N(\gamma))+J^2(J+Y_N(\gamma))} = o(\gamma), \end{aligned} \tag{24}$$

from which we can derive how small departures of γ from zero due to marginal price discrimination affect the arrival rate, J. Specifically, we have that

$$\frac{\mathrm{d}G(E,0)}{\mathrm{d}\gamma} = \frac{EX^4}{(E+X)(E^2+2EX+2X^2)^2},$$

$$\frac{\mathrm{d}G(E,0)}{\mathrm{d}J} = \frac{\int_\tau A(\tau)\mathrm{d}\tau}{N}\frac{1}{E^2} - \frac{2EX(E+2X)}{(E^2+2EX+2X^2)^2} \underbrace{=}_{G(E,0)=0} \frac{2X^2(E^2+4EX+2X^2)}{E(E^2+2EX+2X^2)^2} > 0.$$

But then, the effect of a price discrimination on arrival rate J is equal to

$$\frac{\mathrm{d}J}{\mathrm{d}\gamma} = -\frac{\frac{\mathrm{d}G(E,0)}{\mathrm{d}\gamma}}{\frac{\mathrm{d}G(E,0)}{\mathrm{d}J}} = -\frac{E^3X}{(E^2+2EX+2X^2)(E^2+4EX+2X^2)} < 0. \tag{25}$$

The interpretation of this result is the following. With exogenous entry decisions, maximization of the social welfare amounts to minimization of the search costs of the entering motorists. Because the fraction of entering economy-strategy motorists is negligibly small, the objective of the social planner is then to minimize the search cost of the fast-strategy motorists, equal to (cf.(17))$NE(\epsilon) = \int_\tau A(\tau)\mathrm{d}\tau \frac{1}{1-q_{11}(\epsilon)}$. A marginal change in the price difference ϵ and the associated change in γ, as we show in (25), reduces such search cost, and thereby increases social welfare.

What if the sensored parking was made more expensive? Then, the conclusion of this section is reversed. Economy-strategy motorists are still those above some parking-duration cutoff τ^*, but they now search for non-sensored, rather than sensored, parking. The relation of the exit rates X_N and X_S to the average X is now reversed: X_S rises above X while X_N becomes smaller. The derivative $\frac{\mathrm{d}G(E,0)}{\mathrm{d}\gamma}$ becomes negative, meaning that $\frac{\mathrm{d}J}{\mathrm{d}\gamma} > 0$, i.e. the rise in the mass of economy-strategy parked vehicles increases the aggregate search time and thereby reduces welfare.

5. Second sensor per set?

Consider a government that has already equipped one of two parking bays in a set with a sensor. Does it make sense to equip the second bay with a sensor, as well?

5.1. Uniform pricing

In the environment of uniform pricing where all parking costs the same, all motorists use the fast search strategy. When both bays are equipped with sensors, the only modification of the fast search strategy is that a motorist can randomize when both bays are vacant. Suppose, in such event, the probability of taking the "first" bay (e.g. that on North-South street) is $f \in [0,1]$, so the "second" (e.g. that on East-West street) is taken with probability $1-f$. Then, the condition of equilibrium in transition between states is modified from (7) as follows:

$$\begin{pmatrix} -E & X & X & 0 \\ (1-f)E & -E-X & 0 & X \\ fE & 0 & -E-X & X \\ 0 & E & E & -2X \end{pmatrix} \begin{pmatrix} q_{00} \\ q_{01} \\ q_{10} \\ q_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

which modifies the equilibrium distribution across states (8) as follows:

$$\begin{pmatrix} q_{00} \\ q_{01} \\ q_{10} \\ q_{11} \end{pmatrix} = \frac{1}{(E+X)(E^2+2EX+2X^2)} \begin{pmatrix} 2X^2(E+X) \\ EX(2X(1-f)+E) \\ EX(E+2Xf) \\ E^2(E+X) \end{pmatrix},$$

and therefore the congestion q_{11} is unchanged from a single-sensor case. Thus, we can conclude that a second sensor does not expedite search in case of uniform pricing, and therefore has zero economic value. Intuitively, even with one sensor a motorist will not park only if both bays are occupied, hence one sensor is enough to reduce the search time to the minimum.

5.2. Price discrimination

Consider price discrimination with the “first” bay being equipped with a sensor and being cheaper, as prescribed by Section 4.2. How does a second sensor alter the equilibrium? Consider first how payoffs to fast and economy search strategies are changed.

The fast strategy prescribes to take the first available bay. If both bays are vacant, a motorist would obviously select the cheaper one. But the cheaper bay is the one initially equipped with a sensor, hence the fast strategy of search does not change when the second sensor is installed.

The economy strategy is to look for a cheaper bay. Because the second sensor does not alter prices, it does not alter the economy strategy, too.

Because the search strategies are unchanged, in equilibrium the probability distribution of occupancy states, as well as the choice of a strategy by motorists, will remain unchanged. Under price discrimination, a second sensor has no value, too.

6. The effect of price regulation on sensor installation incentives

Demand for parking is greatly affected by the price for parking. Many cities around the world offer parking that is too cheap, which leads to excess demand and shortage of available parking space. Low price also leads to cruising for parking, i.e. driving around the destination location and searching for vacancy.[3] To address the problem, such cities often increase the supply of parking, either by investing public funds or by mandating private landowners to supply a certain amount of parking. Shoup (2005) provides an extensive account of social costs caused by excess incentives to build parking, that are in turn caused by suboptimal price.

This section investigates how the incentives to install occupancy sensors are affected by price regulation, particularly in comparison with the incentives to build additional parking. To keep variation in such regulation unidimensional, we will assume that pricing is uniform, i.e. all parking costs an exogenous amount of p per unit of time. Because the primary role of price regulation is to influence the decision to search for parking, we return to the initial assumption that entry decisions are endogenous, as described in Section 3.1. The government can improve each set of two parking bays in two different ways, which results in three types of parking sets:

[3] There is a substantial literature that addresses the social costs of cruising for parking. Liu and Geroliminis (2016) is a recent theoretical analysis. Van Ommeren et al. (2011) assess the magnitude of the problem empirically. Inci (2015) provides further references on the problem.

Unimproved set The unimproved set consists of two parking bays without sensors, as described in Section 2.1. Motorists randomly decide which of the two bays to inspect for vacancy, each with equilibrium probability of $\frac{1}{2}$. With arrival intensity per set of E and exit intensity of X, the congestion level, i.e. the probability that the inspected bay is occupied, is $\frac{\frac{1}{2}E}{\frac{1}{2}E+X}$.

Intensive improvement The intensive improvement consists of installing a sensor in one of two bays. Because pricing is uniform, all motorists follow the fast strategy of search. The congestion level, i.e. the probability that both bays are occupied, is given by (9). The share of all parking sets improved in this way is denoted α_S.

Extensive improvement We assume that the government can potentially build an third parking bay in a set. The spatial interpretation of Figure 1 can be modified as follows: assume an extra parking bay was added to one of the two streets, e.g. on the East-West street. A motorist can then decide whether to inspect the street with one bay (North-South) or with two bays (East-West). In equilibrium, motorists randomize with probabilities $\frac{1}{3}$ and $\frac{2}{3}$, respectively. The congestion level is given by $\frac{\frac{1}{3}E}{\frac{1}{3}E+X}$. The fraction of extensively improved parking sets is denoted by α_N.

Denote by $E(p,q)$ the intensity of arrivals per set, given by (5), and by $X(p,q)$ is the intensity of departures from an occupied bay, given by (6). Arrival rate $E(p,q)$ is the same for all three types of parking sets because the sets are drawn randomly rather than selected; exit $X(p,q)$ is the same, too, because the search strategy is the same for motorists of all types so the probability of landing at a set of particular type does not depend on τ or v. Also denote by $R(p,q)$ the ratio of the two,

$$R(p,q) \equiv \frac{E(p,q)}{X(p,q)} = \frac{\int_\tau \int_{v^*(p,q,\tau)}^{\infty} A(\tau,v)\tau \mathrm{d}v \mathrm{d}\tau}{N(1-q)}. \tag{26}$$

Because motorists draw one of three types of parking sets randomly and independently, the equilibrium congestion level q is the chance of not being able to park averaged across the three types of parking:

$$H(p,q) \equiv (1-\alpha_S-\alpha_N)\frac{R(p,q)}{R(p,q)+2} + \alpha_S\frac{R(p,q)^2}{R(p,q)^2+2R(p,q)+2} + \alpha_N\frac{R(p,q)}{R(p,q)+3} - q \equiv 0. \quad (27)$$

The generation of social welfare from the parking process, per unit of time, can be defined as follows:

$$W(p,q) \equiv \int_{\tau}\int_{v^*(p,q,\tau)}^{\infty} A(\tau,v)v\tau \mathrm{d}v\mathrm{d}\tau - cNE(p,q), \quad (28)$$

where the first component is the generation of welfare from parking *per se*, while the second component is the aggregate time opportunity cost of search for parking.

6.1. The relative return to sensored parking

Suppose the government is determined to spend a certain amount of funds on improving parking supply and is choosing between the intensive improvement, i.e. increasing the share of sensored parking α_S, and the extensive improvement, i.e. building additional parking bays and thereby increasing α_N. The choice of such government depends on the comparison of the returns to each type of investment. In particular, it is more likely to choose the intensive improvement when the relative return to such improvement is higher. Therefore, it is important to understand how the price regulation affects such relative return.

The absolute social return to expanding parking of type $i \in \{S,N\}$ can be calculated as follows:

$$\frac{\mathrm{d}W}{\mathrm{d}\alpha_i} = \frac{\partial W}{\partial q}\frac{\partial q}{\partial \alpha_i} = -\frac{\partial W}{\partial q}\frac{\frac{\partial H}{\partial \alpha_i}}{\frac{\partial H}{\partial q}}. \quad (29)$$

The relative social return can then be calculated as follows:

$$\frac{\frac{\mathrm{d}W}{\mathrm{d}\alpha_S}}{\frac{\mathrm{d}W}{\mathrm{d}\alpha_N}} = \frac{\frac{\partial H}{\partial \alpha_S}}{\frac{\partial H}{\partial \alpha_N}} = \frac{\frac{R^2}{R^2+2R+2} - \frac{R}{R+2}}{\frac{R}{R+3} - \frac{R}{R+2}} = \frac{2(R+3)}{R^2+2R+2}. \tag{30}$$

Proposition 1. *The relative social return to sensored parking, given by (30), increases with the price for parking p.*

Using the language of International Economics, a higher price for parking increases the comparative advantage of investment into sensors over the investment into extra parking space.

Proof. Throughout the proof, we treat the congestion q as a function of the arrival-exit ratio R, rather than the reverse. The value of $q(R)$ is defined by (27), such that $\frac{\mathrm{d}q}{\mathrm{d}R} > 0$. Rewrite (26) as follows:

$$NR(1 - q(R)) \equiv \int_\tau \int_{v^*(p,q(R),\tau)}^\infty A(\tau, v)\tau \mathrm{d}v \mathrm{d}\tau. \tag{31}$$

From the definition of $q(R)$ in (27), it can be shown that the left-hand side of (31) is increasing in R. At the same time, the right-hand side of (31) is decreasing in both p and R. But that means that the arrival-exit ratio decreases with the price, $\frac{\mathrm{d}R}{\mathrm{d}p} < 0$. The comparative return to sensored parking (30) is a decreasing function of R, hence a higher price for parking p increases such comparative return via lowering R. ∎

6.2. The absolute return to sensored parking

According to (30), a very high arrival-exit ratio R that may be associated with free parking will make the comparative return to sensor installation close to zero. But what about the absolute return to sensors? Is it also converging to zero with congestion, or remains substantial? Understanding the absolute welfare gains from extra sensors may be important for a government that does not consider building extra parking space, e.g. due to prohibitively high cost of doing so. To remove unnecessary complexity from the analysis, we make the following simplifying assumptions. The share α_N of extensively improved parking is zero. Parking is free, $p = 0$, which corresponds to maximum congestion, and is the most interesting case to analyze. Instead of analyzing the full range of $\alpha_S \in [0, 1]$, we focus on the two corners: $\alpha_S = 0$ when no parking is sensored, and $a_S = 1$ when all parking sets have one sensor.

Suppose that when $\alpha_S = i$, $i \in \{0, 1\}$, the occupancy rate (2) is given by some $Q_i = 1-\mu_i$. To model high congestion, we will consider μ_i being arbitrarily close to zero. Section 3.2 demonstrates that, without sensors, congestion q_0 equals occupancy Q_0, hence the expected search time without sensors is $\frac{1}{1-q_0} = \frac{1}{\mu_0}$. When sensors are installed, the new congestion level is given by (10) hence the new expected search time is

$$\frac{1}{1-q_1} = \frac{1}{\frac{1}{2} - Q_1 + \frac{1}{2}\left(4Q_1(1-Q_1)+1\right)^{\frac{1}{2}}} = \frac{1}{2\mu_1} + o(\mu_1). \tag{32}$$

If entry is exogenous, installation of sensors does not change occupancy, hence $Q_0 = Q_1$, $\mu_0 = \mu_1$, and therefore sensors cut the search time in half. The intuition is similar to that provided at the end of Section 3.3.1: if one bay in a set is vacant, a sensor increases the probability of finding that bay from 50% to 100%.

With endogenous entry, reduction in the search time will induce additional entry, offsetting part of the search time reduction. But such reduction will still remain substantial, leading to a non-trivial decrease in the second (negative) component on the right-hand side of (28). Moreover, extra entry means extra utility from parking process, i.e. the first component on the right-hand side of (28) will be increased.

Thus, we can conclude, the absolute welfare gains of sensor installation in extremely congested conditions can be substantial.

7. Conclusion

This paper is the perhaps the first attempt to look into the economic merits of parking occupancy sensors. It modifies the "binomial approximation" model of search for parking to introduce elements of directed search that may be affected by information from the sensors. The research questions of the study can be broadly divided into two parts: how installation of sensors affect pricing, and how pricing affects incentives to install sensors.

We find that installation of sensors changes the optimal price in a non-monotone way. In a more interesting case of highly congested parking, sensors increase the externality of a parked vehicle on those searching for parking, hence increase the optimal price. We also find that, if price discrimination is considered, sensored parking should be made cheaper than non-sensored one. This would attract long-term parkers to the sensored bays, that are naturally more occupied, and relieve the non-sensored bays for short-term parkers who value short search durations.

This paper finds that it is not necessary to saturate all parking with sensors. Increasing the fraction of sensors beyond some level, one half in our stylized model, does not expedite search and thus has no economic value. The paper also finds that too cheap parking and associated excessive congestion dampens the incentive to install sensors, relative to the incentive to build more parking. At the same time, in absolute terms the return to sensors remains significant even during high congestion.

The theoretical analysis of this paper has drawbacks that are shared by virtually all theoretical models of search for parking: it ignores (most of) spatial correlation of parking bay occupancy. While Arnott and Williams (2017) simulate such correlation in a model with non-sensored parking, further research is needed to understand how sensors change the game. Also, the current paper assumes that motorist behavior can be affected only by the nearest sensor, while the real search strategies can be more complex, and perhaps can only be analyzed in a simulated rather than analytical model.

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